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Lava channel formation via the viscoplastic indentation of hot substrates

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[1] Lava channels that appear to have been incised into the underlying substrate are commonly ascribed to thermal or thermomechanical erosion. Here we show that where the associated lava flow moves on a hot and soft substrate, the channel may also be ascribed to viscoplastic indentation of the substrate. A simple model of viscoplastic indentation leads to distinct predictions that can be readily tested in the field. For example, two broad, wedge-shaped, levee-like features should rise from the substrate during indentation. Such features appear to have risen alongside of a channel that was incised into a hot and soft, recently emplaced basaltic substrate during the 2001 eruption on Mount Etna.
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1. Introduction

[2] A typical lava channel on Mount Etna is flanked by a pair of mutually parallel, narrow levees [*Sparks et al.*, 1976]. These levees are constructional, i.e., they self-assemble on the substrate on which the lava flows, along the longitudinal margins of the lava flow, where intense heat losses and a sluggish flow promote a rapid solidification of the lava [*Sparks et al.*, 1976; *Hulme*, 1974]. Figure 1a shows several typical lava channels with their pairs of constructional levees criss-crossing one another on a slope on Mount Etna. Figure 1a also shows a channel that is distinctly different from the others. This channel (marked "IC" in Figure 1a) appears to have been incised into the substrate [*Ferlito and Ori*, 2002; *Ferlito and Siewert*, 2006]. Figures 1b and 1c show two alternative views of the incised channel. Incised lava channels are rare on Earth [*Greeley et al.*, 1998], but they might be common on other planets [*Hulme*, 1973; *Head and Wilson*, 1981; *Gregg and Greeley*, 1993; *Wilson and Mouginis-Mark*, 2001].

[3] The incised channel of Figure 1 as well as the unusual circumstances that led to its formation have been described by *Ferlito and Ori* [2002]. The channel was incised by a large (up to 20 m thick) lava flow that originated at the base of the "Laghetto" scoria cone [*Calvari and Pinkerton*, 2004] and flowed for 12 hours during the 2001 eruption on Mount Etna.

[4] This large lava flow moved 2 km down a gently sloping (5 to 10°) substrate. Yet the large lava flow incised a channel only in the proximal 220 m of its course, where several smaller lava flows had emplaced a layered basaltic substrate within 5 days prior to the large lava flow [*Ferlito and Ori*, 2002; *Ferlito and Siewert*, 2006]. Each one of these smaller lava flows was up to 500 m long. The total number of smaller lava flows (and therefore the total thickness of the layered basaltic substrate) has not been reported. Nevertheless, the banks of the incised channel showed evidence of at least 11 smaller lava flows, in the form of 11 layers of the basaltic substrate, each layer up to 30 cm thick [*Ferlito and Siewert*, 2006].

[5] When the smaller lava flows ceased and the large lava flow started to incise a channel into the layered basaltic substrate, the oldest layers of the substrate were 5 days old. The substrate had been emplaced fast, and it must have been hot (*Ferlito and Siewert* [2006] estimate $T \approx 900^\circ$C), albeit not uniformly hot due to an incomplete thermal equilibration.

[6] Downstream of the proximal 220 m of its course, the large lava flow did not incise a channel. Instead, it formed a channel flanked by constructional levees (Figure 1). Thus, the incised channel was 220 m long. It was 12 m wide and up to 6 m deep.

2. Abrasive Wear Mechanism

[7] In a recent paper, *Ferlito and Siewert* [2006] speculate on the mechanism that might have been responsible for the formation of Mount Etna's incised channel (Figure 1). After showing conclusively that the rate of heat transfer was insufficient to form the channel by thermal erosion (i.e., by melting of the substrate to the depth observed [*Kerr*, 2001; *Fagents and Greeley*, 2001]), *Ferlito and Siewert* [2006] propose that the channel might have been formed by plastic abrasive wear. In the model of plastic abrasive wear, the lava flow slides over the substrate, removing material from the substrate and carrying it downstream. The abrasive wear can be effective only where the basaltic substrate is hot, and the yield stress of the substrate accordingly low. Thus, the model of abrasive wear accounts for the fact that the incised channel was only 220 m long. The model also accounts for the field observation that the banks of the channel exposed the layers of the substrate [*Ferlito and Ori*, 2002; *Ferlito and Siewert*, 2006].

[8] To test quantitatively the model of abrasive wear, *Ferlito and Siewert* [2006] estimate the yield stress of the hot ($T \approx 900^\circ$C) substrate and the sliding distance; then, they use Archard's theory of plastic abrasive wear [*Archard*, 1953] to infer a value of wear coefficient, $k \approx 10^{-3}$, that falls within the range of values (10^{-1} to 10^{-5}) that has been measured in tribology [*Ferlito and Siewert*, 2006]. In view of the wide range of likely values of k, a more stringent test of the model appears to be unfeasible. Nevertheless, we may yet question

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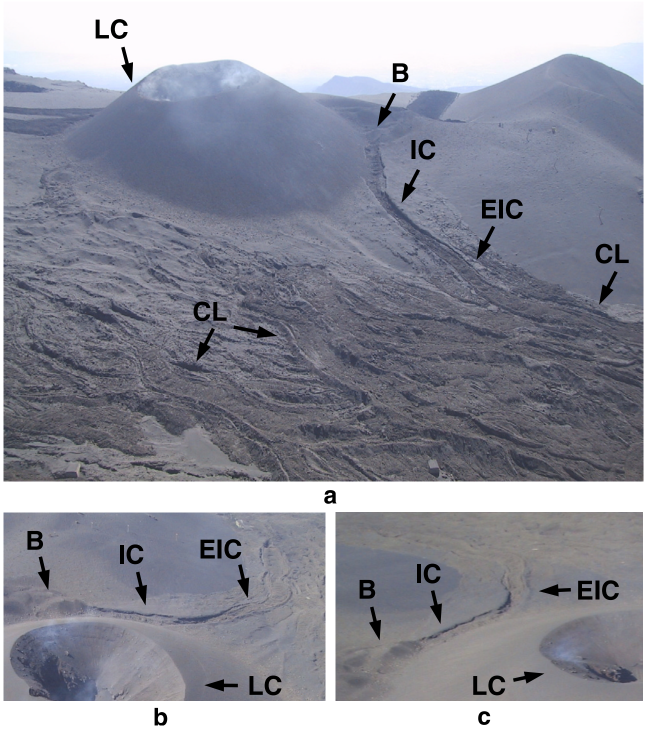

Figure 1. Three photographs showing different views of the incised channel (marked "IC"), the bocca (marked "B") at the source of the incised channel, the endpoint of the incised channel (marked "EIC"), the "Laghetto" cone (marked "LC"), and the surrounding slope on Mount Etna. The distance from B to EIC is the length of the incised channel, about 220 m. Downstream of EIC, the channel continues in the form of a typical lava channel flanked by constructional levees (marked "CL"). (a) View from the downstream end of the incised channel; this view also includes several typical lava channels flanked by constructional levees. (b) View from the Laghetto cone. (c) View from the upstream end of the incised channel. Photographs courtesy of Sonia Calvari, Istituto Nazionale di Geofisica e Vulcanologia, Catania, Italy; used with permission.

the sliding distance estimated, s = 2 km. In plastic abrasive wear, s is the relative displacement between the asperities of the abrasive material and the plastic substrate where these asperities embed themselves and scratch grooves [*Archard*, 1953; *Burwell and Strang*, 1952]. (The lavas of Mount Etna are extruded with a relatively high fraction of solid crystals. Clots of these solid crystals might have acted as the asperities of the abrasive lava flow.) Thus, for a 2 km-long lava flow, the estimate s = 2 km corresponds to an extreme and perhaps improbable scenario: a plug lava flow with uniform velocity across its thickness and a sharp shear layer at its base. Besides plastic abrasive wear, other mechanisms might have contributed to the formation of the incised channel.

3. Viscoplastic Indentation Mechanism

[9] Where the substrate was hot, the channel might have been formed, at least in part, by viscoplastic indentation of the substrate, as sketched in Figure 2. In the idealized model of Figure 2, the lava flow of thickness h and width w acts as a heavy, flat, long rigid punch that presses downwards on the underlying substrate. Under pressure from the weight of this punch, the substrate flows viscoplastically (and under plane-strain conditions); as a result, the punch cuts vertically into the substrate, forming an imprint or indentation of the same width as the punch—the incised channel of width w. (A similar indentation model has been proposed for the tectonic collision of India—the punch—with Eurasia—the substrate [*Tapponniere and Molnar*, 1976; *Molnar and Tapponniere*, 1977].)

[10] At any given time t during viscoplastic indentation, the state of the deforming substrate is defined by the vertical, downward-cutting displacement of the punch or lava flow, $u = u(t)$ (Figure 2). Since the viscoplastic flow is incompressible, as u increases the area swept by the punch must be compensated by the extrusion of two wedge-shaped, levee-like features of width w and height u (Figure 2). (In a similar way, during the tectonic collision of India with Eurasia, the area swept by India was compensated by the extrusion of Indochina towards the Southeast [*Tapponniere and Molnar*, 1976; *Molnar and Tapponniere*, 1977].) The rise of these levee-like features entails the rise of the banks of the channel with a displacement u upwards from the initially level surface of the substrate (Figure 2). Thus, the total depth of the incised channel measured from the upper rim of the banks is $d = 2u$.

[11] The energy equation that governs the indentation is

$$\rho g h w \dot{u} = \tau \dot{\gamma} w^2 + \rho g u w \dot{u}, \qquad (1)$$

where ρ is the density of both the lava flow and the substrate, g is the gravitational acceleration, $\dot{u} = du(t)/dt$, τ is the mean stress in the substrate, $\dot{\gamma}$ is the mean viscoplastic strain rate in the substrate, $\tau\dot{\gamma}$ is the mean viscoplastic power per unit volume, and w^2 is an estimate of the volume of substrate (per unit length of channel) that flows viscoplastically. The left-hand side of Equation (1) is the gravitational energy per unit time that becomes available as the punch or lava flow cuts vertically downward; the first term on the right-hand side is the energy per unit time required to make the substrate flow viscoplastically; and the second term on the right-hand side is

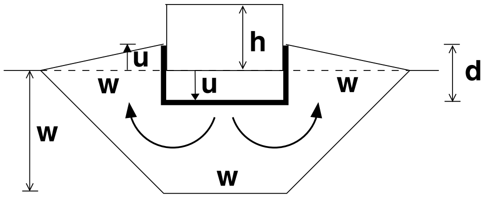

Figure 2. Plane-strain model of channel formation by viscoplastic indentation. The dashed line represents the initially level surface of the substrate. The long punch or lava flow of thickness h and width w is shown in cross section. The downward-cutting displacement of the punch is u whereas the area swept by the punch is uw. The incised channel (of width w) runs normal to the figure; the surface of the channel is the thick, U-shaped line. The curved arrows indicate the viscoplastic flow in the substrate. The slanted lines represent the upper surfaces of two broad, wedge-shaped, levee-like features that rise from the substrate during indentation. The area of these levee-like features equals the area swept by the punch, uw; thus, the viscoplastic flow in the substrate is volume-preserving, and the total depth of the channel is $d = 2u$.

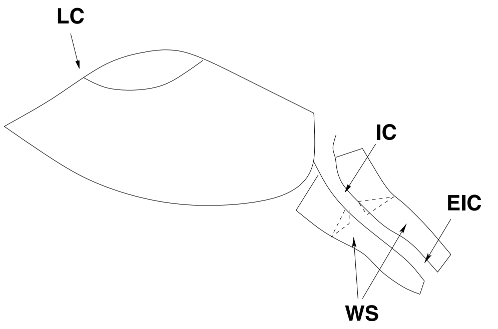

Figure 3. An outline (marked "WS") of the wedge-shaped, levee-like features discernible in Figure 1a. Same as in Figure 1a, the "Laghetto" cone is marked "LC", the incised channel is marked "IC", and the endpoint of the incised channel is marked "EIC."

the gravitational energy per unit time required to raise the banks of the channel and form the levee-like features of Figure 2.

[12] To turn the energy equation (1) into an ordinary differential equation (ODE) for $u(t)$, we estimate $\dot{\gamma} = \dot{u}/w$ and write $\tau = \tau_0 + \eta\dot{\gamma}$ (the constitutive equation of a Bingham fluid), where τ_0 is the yield stress and η the plastic viscosity of the substrate. (More precisely, for a Bingham fluid $\dot{\gamma} = (\tau - \tau_0)/\eta$ if $\tau > \tau_0$ and $\dot{\gamma} = 0$ otherwise.) Substituting these expressions for τ and $\dot{\gamma}$ in the energy equation, solving the ensuant ODE for u (with the initial condition $u = 0$ for time $t = 0$), and taking into account that $d(t) = 2u(t)$, we obtain

$$d(t) = d'[1 - \exp(-t/t')], \tag{2}$$

where $d' \equiv 2(h - \tau_0/\rho g)$ and $t' \equiv \eta/\rho g w$. Thus the depth of the incised channel, $d(t)$, increases monotonically in time and tends to d' as $t \to \infty$. For the indentation to cease at a given limiting depth d', it must be that $\tau_0 = \rho g(h - d'/2)$. Using $h = 10$ m (the mean thickness of the lava flow measured in the field [*Ferlito and Siewert*, 2006]), $d' = 4$ m (the mean depth of the channel measured in the field [*Ferlito and Siewert*, 2006]), and $\rho = 2800$ kg/m^3 (the density used by *Ferlito and Siewert* [2006]), we infer a value of yield stress, $\tau_0 = 0.2$ MPa, that falls within the range of likely values (0.1 to 1 MPa) for a substrate at about 900°C [*Ferlito and Siewert*, 2006]. Further, the time required for $d(t)$ to attain a value within 10% of the limiting depth d' is $t \approx 2\,t'$. Using $w = 12$ m (the mean width of the channel measured in the field [*Ferlito and Siewert*, 2006]) and $\eta = 10^9$ Pa s, we estimate $t' = 2$ hr, or about one-fifth of the duration of the lava flow. (The viscosity of basalt rises sharply at a midpoint between the liquidus T and the solidus T, and then slowly as the temperature drops below the solidus T. The viscosity may be as low as 10^3 Pa s at the solidus T, and slightly higher at lower T, but it increases by orders of magnitude in a few days [*McBirney and Murase*, 1984]. Note that the apparent viscosity, $\eta_a \equiv \eta + \tau_0/\dot{\gamma}$, becomes much higher than the viscosity η for $t > t'$ [*Griffiths*, 2000].)

4. Discussion

[13] The geometry of the model of viscoplastic indentation of Figure 2 is defined by a single length scale. In fact, the depth to which the substrate flows viscoplastically (hereafter to be denoted d_f), the width of the levee-like features predicted by the model (hereafter to be denoted w_l), and the width of the channel (w) are linked by the relations $d_f \approx w_l \approx w$. These relations are predicated on the assumption that $d_s \geq w$, where d_s denotes the depth to which the substrate is hot and capable of flowing viscoplastically. (Note that it must always be that $d_s \geq d_f$.) Were it the case that $d_s < w$, then it would have to be that $d_f \approx d_s$, and the relation $d_f \approx w_l$ would still be appropriate, but there would be two independent length scales, w on the one hand and $d_s \approx d_f \approx w_l$ on the other, and these independent length scales would be linked only by the relations $d_s \approx d_f \approx w_l < w$.

[14] The levee-like features predicted by the model of Figure 2 appear to be discernible in the photograph of Figure 1a, in the form outlined in Figure 3. Further, from Figure 1a and Figure 3, it appears that $w_l \approx w$, in keeping with the model of Figure 2. Thus, even though there is no direct field evidence that on Mount Etna $d_s \geq w$, from Figure 1a and Figure 3, which indicate that $w_l \approx w$, and from the fact that the relations $d_f \approx w_l$ and $d_s \geq d_f$ must always be satisfied, we infer that on Mount Etna it must have been that $d_s \geq w$, as required by the model.

[15] Note that in the model of viscoplastic indentation the banks of the channel are the surfaces of vertical cuts through the layers of the substrate (Figure 2). Each one of these cuts may be thought of as the strike of a transform fault in which the displacement parallel to the fault is discontinuous across the strike of the fault. Nevertheless, a transform fault is not merely the streak of the fault, and the displacement associated with a transform fault is not limited to the relative displacement between the two sides of the strike of the fault. Instead over a narrow (but finite) band that straddles the streak of the fault, the displacement parallel to the fault changes smoothly but substantially, adding much to the relative displacement across the fault. In a similar manner, over a narrow (but finite) band that straddles each one of the vertical cuts shown in Figure 2, the displacement parallel to the cut changes

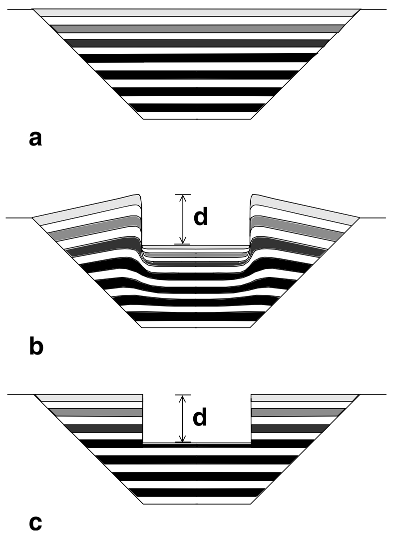

Figure 4. Geometry of the layers of the substrate (a) before the channel is incised, (b) after viscoplastic indentation, and (c) after abrasive wear. (Schematic.) The layers are shown in different shades of gray to facilitate the visualization.

smoothly but substantially. As a result, the layers of the substrate (Figure 4a) should bend downward close to the banks of the channel (Figure 4b). This prediction of the viscoplastic indentation model is in accord with field observations [*Ferlito and Ori*, 2002].

5. Conclusions

[16] There are notable differences between the model of viscoplastic indentation and the model of abrasive wear. In the model of viscoplastic indentation, basalt is squeezed out from the bulk of the substrate below the lava flow. The squeezed-out basalt migrates laterally with the viscoplastic flow in the substrate. As a result, the basalt on the floor of the channel (Figure 4b) coincides with the basalt that lied on the surface of the substrate before the channel was incised (Figure 4a). In addition, two broad, wedge-shaped, levee-like features rise alongside of the channel, and the layers of the substrate bend downward close to the banks of the channel (Figure 4b).

[17] In the model of plastic abrasive wear, basalt is scratched off the surface of the substrate. The scratched-off basalt migrates downstream with the lava flow. As a result, the basalt on the floor of the channel (Figure 4c) coincides with the basalt that lied buried at a depth d within the substrate before the channel was incised (Figure 4a). In addition, the banks of the channel remain level with the surface of the substrate, and the layers of the substrate remain horizontal close to the banks of the channel (Figure 4c).

[18] On the other hand, both models require a substrate with a low yield stress; thus, both models explain the fact that the channel was incised only where the substrate was hot [*Ferlito and Siewert*, 2006]. Also both models predict that the banks of the channel should expose the layers of the substrate (Figures 4b and 4c), in accord with field observations [*Ferlito and Siewert*, 2006].

[19] A channel is likely to be incised into a hot substrate by the concurrent action of viscoplastic indentation and plastic abrasive wear. For any given channel, we will be able to identify the leading mechanism by comparing field observations with the disparate predictions discussed above.

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