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Review Report

Kinematics of Local Vortex Identification Criteria

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1. Introduction

Since Leonardo da Vinci's famous sketches of vortices in turbulent flows, fluid dynamicists for over five centuries have continued to interpret turbulent flows in terms of motion of vortices. Nevertheless, much debate surrounds the question of how to unambiguously define vortices in turbulent flows (for example, refer to Jeong and Hussain, 1995; Chakraborty et al., 2005). In the last few decades, technological advances have resulted in a wide availability of high-resolution experimental and numerical data of turbulent flows, thereby prompting the need of visualizing vortices in these flows. The problem of visualizing the vortical regions in turbulent flows hinges upon having a mathematical criterion—a vortex-identification criterion—for defining these regions.

2. Characteristics of a Vortex

In this review, we restrict attention to filamentary shaped vortices (as opposed to three-dimensional blobs or two-dimensional sheets) and to local or point-wise vortex-identification criteria. A vortex identification criterion prescribes a set of characteristics that a vortex should satisfy. In ideal fluids the existence of a sharp boundary between rotational and irrotational fluid results in an unequivocal definition of a vortex filament (Saffman, 1992). In real fluids, however, the diffusion of vorticity by viscosity prohibits the possibility of such a crisp definition. In turbulent flows, there is no universally accepted set of characteristics for a vortical region. This lack of agreement has resulted in a plurality of vortex-identification criteria—each criterion ascribing a different set of characteristics that a vortex should satisfy. (The reader is referred to Jeong and Hussain 1995 for a discussion on the inadequacies of common intuitive measures of detecting vortices--local pressure minima, closed or spiraling streamlines and pathlines, and iso-vorticity surfaces.)

We adopt the framework of critical-point theory, where a vortex satisfies the following kinematic characteristics of local streamlines when viewed from a frame of reference translating with a fluid particle inside the vortex (Chakraborty et al., 2005):

(i) the local flow is swirling;
(ii) the separation between the swirling material points inside the vortex remains small, i.e., the orbits of the material points are compact.

Next we summarize the commonly employed local vortex-identification criteria for incompressible flows and their relation with the abovementioned characteristics of a vortex.

3. Local Vortex-Identification Criteria

The Q criterion identifies vortices as flow regions with positive second invariant of the velocity gradient tensor, *grad v*, i.e., $Q > 0$, and Q can be written as (Hunt et al., 1988):

$$Q = \frac{1}{2}\left(\| \Omega \|^2 - \| S \|^2\right), \tag{1}$$

where, Ω and S are the skew-symmetric and symmetric parts of *grad v*, respectively. Hence, Q is a local measure of excess rotation-rate relative to strain-rate.

The λ_2 criterion is based on a modification of the concept of vortices being associated with local pressure minimum in a plane (Jeong and Hussain, 1995). The λ_2 criterion defines the vortex as a connected region where the intermediate eigenvalue (λ_2) of the symmetric tensor, $S^2 + \Omega^2$, is negative. The connection between the requirement $\lambda_2 < 0$ and local pressure minimum is debatable (Chakraborty et al., 2005).

The Δ, λ_{ci}, and enhanced swirling strength criteria are based on critical-point theory. In the region where the eigenvalues of *grad v* are complex, there exists a plane where the local streamlines are swirling, and are characterized by following two kinematic parameters (Chakraborty et al., 2005):

(i) λ_{ci}, the imaginary part of the complex eigenvalue, is a measure of the strength of swirling. In the plane of swirling, $2\pi/\lambda_{ci}$ is the time for one complete revolution.

(ii) $\lambda_{cr}/\lambda_{ci}$, the ratio of the real to imaginary part of the complex eigenvalue pair, is a measure of the compactness in the spiraling orbits. During one complete revolution in the plane of swirling, $exp(2\pi\lambda_{cr}/\lambda_{ci})$ is the ratio of the final to initial radial positions. The sign of the ratio indicates the nature of spiraling: positive indicates outward and negative indicates inward spiral.

This characterization is a consequence the requirements for a vortex discussed in the previous section. The plane of swirling is identified to be the plane spanned by the real and imaginary components of the complex conjugate eigenvectors. The two parameters, λ_{ci} and $\lambda_{cr}/\lambda_{ci}$, have unambiguous interpretation in terms of the local flow kinematics: they measure the swirling strength and the spiraling compactness, respectively.

The Δ criterion (Chong et al., 1990) defines the vortex core to be the region where *grad v* has complex eigenvalues. This region is identified by positive values of Δ, the discriminant of the characteristic equation of *grad v*. The swirling strength criterion (Zhou et al., 1999) uses $\lambda_{ci} > \varepsilon$ to identify vortices (ε is a positive threshold). Although for a zero threshold, $\Delta = 0$ and $\lambda_{ci} = 0$ are equivalent, they are different for non-zero thresholds (Chakraborty et al., 2005). The enhanced swirling strength criterion (Chakraborty et al., 2005) uses both λ_{ci} and $\lambda_{cr}/\lambda_{ci}$ to identify vortices: the flow should have sufficient local swirling ($\lambda_{ci} > \varepsilon$) and the material points comprising the vortex should be spatially compact ($\lambda_{cr}/\lambda_{ci} < \delta$). The thresholds for λ_{ci} and $\lambda_{cr}/\lambda_{ci}$ are chosen based on the time and length scales of the problem, respectively.

4. Unified Framework for the Local Vortex Identification Criteria

Here, in a unified framework, we inter-relate and interpret the Q, λ_2, and Δ criterion in terms of the two kinematic parameters: λ_{ci} and $\lambda_{cr}/\lambda_{ci}$ (Chakraborty et al., 2005).

The Q criterion can be expressed as:

$$Q = \lambda_{ci}^2 \left(1 - 3\left(\frac{\lambda_{cr}}{\lambda_{ci}}\right)^2\right). \tag{2}$$

The Δ criterion can be expressed as:

$$\Delta = \frac{\lambda_{ci}^6}{27}\left[1 + 9\left(\frac{\lambda_{cr}}{\lambda_{ci}}\right)^2\right]^2. \tag{3}$$

In general, the λ_2 criterion cannot be expressed in solely terms of the two kinematic parameters. In the special case where *grad v* is a normal tensor, λ_2 can be expressed as:

$$\lambda_2 = \lambda_{ci}^2\left(\left(\frac{\lambda_{cr}}{\lambda_{ci}}\right)^2 - 1\right). \tag{4}$$

Using these equations one can interpret imposing thresholds on these criteria in terms of choosing thresholds on the two kinematic parameters. Inside the intense vortical regions in many turbulent flows, it is observed that $\lambda_{cr}/\lambda_{ci}$ is approximately zero, thereby indicating that the local motion is approximately two dimensional (Chakraborty et al., 2005). This property can be used to choose appropriate thresholds—'equivalent thresholds'—for the different criteria for a given the threshold for λ_{ci}. These equivalent thresholds in turbulent flows result in remarkably similar vortices for the different criteria (Chakraborty et al., 2005).

5. Vortex Interactions

The different local vortex-identification criteria discussed above are formulated without any explicit consideration for the effects of vortex interactions on identifying a vortex. Nevertheless, these criteria are frequently employed in visualizing vortices in turbulent flows, where the vortices are often curved and in close vicinity of each other. These scenarios, namely the curvature of vortex axis and the crowding of vortices, result in vortex interactions that affect the local flow. In such flows using a local vortex identification criterion may result in significant mis-characterization of the vortex geometry (Chakraborty et al., 2007). For example, consider the flow constructed by superposition of velocity fields of two co-rotating Gaussian vortices, each having the same vorticity distribution and unit radius. For this flow, the educed vortex shapes are depicted in Fig. 1 for increasing separation between the vortex centers, starting with the vortices just touching each other. In Fig. 2 we show the educed vortex shape for the Gaussian vortices of Fig. 1 whose vortex axes are mutually perpendicular. It is clear that the educed vortex shape for close encounters is misleading.

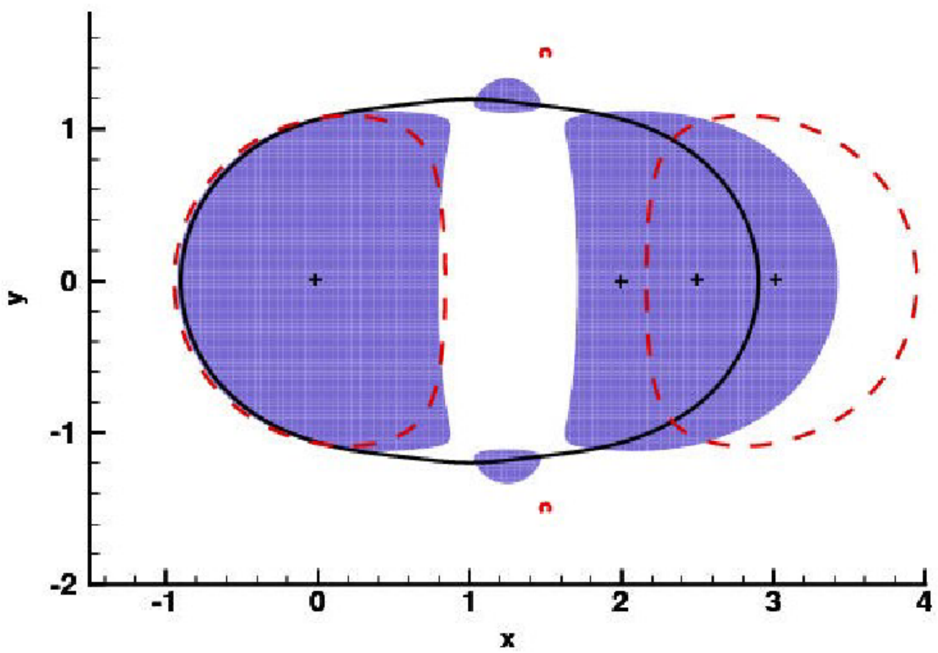

Fig. 1. Vortex shapes for interacting Gaussian vortices. The distance between the vortex centers are: 2 (black line), 2.5 (blue), and 3 (red dashed line). The various local vortex-identification criteria yield the same result because the flow is two dimensional.

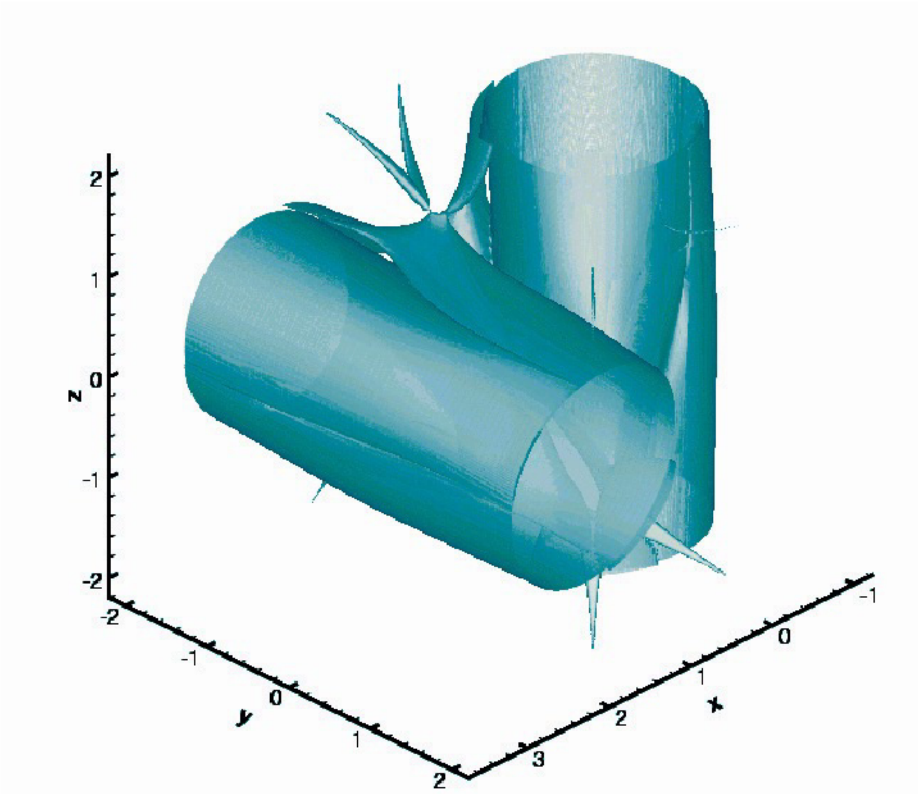

Fig. 2. Vortex shape for interacting Gaussian vortices (using λ_{ci}) with mutually perpendicular vortex axes when the vortices are just touching each other.

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