

Lightcone interactions of massless fields in 4d Minkowski and (Anti-)de Sitter

Yasha Neiman

Okinawa Institute of Science and Technology

2303.17866, 2510.22532, 2606.17380 (with J. Kozaki and J. Lang)

Taiwan Strings, 2026-10-01

Motivation

- 4d Quantum Gravity in finite regions of space
- $\Rightarrow$ Quantum Gravity in dS_4
- $\Rightarrow$ Higher-Spin Gravity (dS_4/CFT_3)
- $\Rightarrow$ Field evolution between an observer's past and future horizons
- $\Rightarrow$ Lightcone formalism for massless fields in $(A)dS_4$

Scope

Massless fields with various spins s in 4d spacetime:

- Yang-Mills: $s = 1$
- General Relativity: $s = 2$
- Higher-Spin Gravity: $s = 0, 1, 2, 3, 4, \dots$

One physical degree of freedom ϕ_h per helicity $h = \pm s$.

Cubic vertices characterized by $h_{\text{tot}} = h_1 + h_2 + h_3$:

- Yang-Mills interactions: $h_{\text{tot}} = \pm 1$
- Gravitational interactions: $h_{\text{tot}} = \pm 2$
- Higher-Spin Gravity: $h_{\text{tot}} = \pm 1, \pm 2, \pm 3, \pm 4, \dots$

Chiral interactions

- Chiral cubic vertices $V_{[3]}$: $h_{\text{tot}} > 0$ (anti-chiral $\bar{V}_{[3]}$: $h_{\text{tot}} < 0$)
- Full theory:

$$\mathcal{L} = \mathcal{L}_{\text{free}} + V_{[3]} + \bar{V}_{[3]} + \text{higher-order vertices}$$

- Chiral theory:

$$\mathcal{L} = \mathcal{L}_{\text{free}} + V_{[3]} \quad \text{and nothing else!}$$

Examples of chiral theories:

- $(h_1, h_2, h_3) = (+1, +1, -1)$: Self-Dual Yang-Mills
- $(h_1, h_2, h_3) = (+2, +2, -2)$: Self-Dual General Relativity
- All (h_1, h_2, h_3) with $h_{\text{tot}} > 0$: Chiral Higher-Spin Gravity

Lightcone gauge / Lightcone formalism

Manifest locality / Lorentz / degrees of freedom:

For spinning massless fields, choose at most 2!

- **Gauge-field Lagrangians:** non-physical degrees of freedom
- **S-matrix:** non-local
- **Lightcone:** broken Lorentz; **foliation along preferred lightlike direction**
 $\ell^\mu = q^\alpha \bar{q}^{\dot{\alpha}}$

Two paths to the lightcone:

- **Lightcone gauge:** start from covariant gauge fields, then specialize.
- **Lightcone formalism:** skip the covariant picture, start from physical degrees of freedom ϕ_h .

Main results

A series of results upgraded from flat ($\Lambda = 0$) to (A)dS ($\Lambda \neq 0$):

- 1 Lightcone gauge for Self-Dual General Relativity [2303.17866]
(Flat result: Plebanski [1975](#))
- 2 Cubic vertices in lightcone formalism for Higher-Spin Gravity
[2606.17380, 2510.22532]
(Flat result: Bengtsson-Bengtsson-Brink [1983](#))
- 3 No higher-order vertices in the chiral theory [2606.17380, 2510.22532]
(Flat result: Metsaev [1991](#))

Lightcone gauge for Self-Dual GR – warmup

Self-Dual Yang-Mills (flat same as (A)dS)

- Potential $A_{\alpha\dot{\alpha}}$ holds $h = +1$, field strength $G_{\alpha\beta}$ holds $h = -1$.
 (“Chalmers-Siegel”, actually Jacobson-Smolín)

$$F_{\alpha\beta} = \partial_{(\alpha}{}^{\dot{\alpha}} A_{\beta)\dot{\alpha}} ;$$
$$\mathcal{L} = G^{\alpha\beta} F_{\alpha\beta} \left(+ G^{\alpha\beta} G_{\alpha\beta} \right)$$

- Lightcone ansatz:

$$A_{\alpha\dot{\alpha}} = q_{\alpha} q^{\beta} \partial_{\beta\dot{\alpha}} \phi_{+1} ;$$
$$\phi_{-1} = q^{\alpha} q^{\beta} G_{\alpha\beta}$$

- Lightcone Lagrangian:

$$\mathcal{L} = \phi_{-1} \square \phi_{+1} + \phi_{-1} (q^{\alpha} \partial_{\alpha}{}^{\dot{\alpha}} \phi_{+1}) (q^{\beta} \partial_{\beta\dot{\alpha}} \phi_{+1})$$

Lightcone gauge for Self-Dual GR – more warmup

Self-Dual GR on Minkowski

- Metric perturbation $h_{\alpha\dot{\alpha}\beta\dot{\beta}}$ holds $h = +2$.
- Weyl curvature $\Psi_{\alpha\beta\gamma\delta}$ holds $h = -2$.
- Lightcone ansatz:

$$h_{\alpha\dot{\alpha}\beta\dot{\beta}} = q_{\alpha} q_{\beta} q^{\gamma} q^{\delta} \partial_{\gamma\dot{\alpha}} \partial_{\delta\dot{\beta}} \phi_{+2} ;$$
$$\phi_{-2} = q^{\alpha} q^{\beta} q^{\gamma} q^{\delta} \Psi_{\alpha\beta\gamma\delta}$$

- Lightcone Lagrangian:

$$\mathcal{L} = \phi_{-2} \square \phi_{+2} + \phi_{-2} (q^{\alpha} q^{\beta} \partial_{\alpha}{}^{\dot{\alpha}} \partial_{\beta}{}^{\dot{\beta}} \phi_{+2}) (q^{\gamma} q^{\delta} \partial_{\gamma\dot{\alpha}} \partial_{\delta\dot{\beta}} \phi_{+2})$$

Lightcone gauge for Self-Dual GR – the result

Covariant formulation of Self-Dual GR with $\Lambda \neq 0$
(Capovilla-Jacobson-Dell)

- Left-handed Lorentz connection $\omega^{\alpha\beta} = \omega_{\mu}^{\alpha\beta} dx^{\mu}$ holds $h = +2$.
- Weyl curvature $\Psi_{\alpha\beta\gamma\delta}$ holds $h = -2$.
- Covariant Lagrangian:

$$F^{\alpha\beta} = d\omega^{\alpha\beta} + \omega^{\alpha\gamma} \wedge \omega_{\gamma}^{\beta} ;$$
$$\mathcal{L} = \Psi_{\alpha\beta\gamma\delta} F^{\alpha\beta} \wedge F^{\gamma\delta} \left(+ \frac{1}{\Lambda} \Psi_{\alpha\beta}{}^{\zeta\xi} \Psi_{\zeta\xi\gamma\delta} F^{\alpha\beta} \wedge F^{\gamma\delta} + \dots \right)$$

Lightcone gauge for Self-Dual GR – the result

- Poincare coordinates $x^\mu = x^{\alpha\dot{\alpha}} = (t, x, y, z)$.
- Reference spacetime: dS $\eta_{\mu\nu}/t^2$ (or AdS $\eta_{\mu\nu}/z^2$).
- Lightcone ansatz:

$$\omega^{\alpha\beta} = \frac{1}{t} dx^{\gamma\dot{\gamma}} \left(2\delta_{\gamma}^{(\alpha} \partial^{\beta)} \dot{\gamma} t + q^\alpha q^\beta q_\gamma q^\delta \partial_{\delta\dot{\gamma}} \phi_{+2} \right) ;$$
$$\phi_{-2} = \frac{1}{t^3} q^\alpha q^\beta q^\gamma q^\delta \psi_{\alpha\beta\gamma\delta}$$

- Lightcone Lagrangian:

$$\mathcal{L} = \phi_{-2} \square \phi_{+2} + t \phi_{-2} (q^\alpha q^\beta \partial_\alpha \dot{\alpha} \partial_{\beta\dot{\beta}} \phi_{+2}) (q^\gamma q^\delta \partial_{\gamma\dot{\alpha}} \partial_{\delta\dot{\beta}} \phi_{+2})$$
$$+ 4 \phi_{-2} (q^\alpha q^\beta \tilde{q}^{\dot{\beta}} \partial_\alpha \dot{\alpha} \partial_{\beta\dot{\beta}} \phi_{+2}) (q^\gamma \partial_{\gamma\dot{\alpha}} \phi_{+2}) ,$$

where $\tilde{q}_{\dot{\alpha}} \equiv q^\alpha \partial_{\alpha\dot{\alpha}} t$.

Main results

A series of results upgraded from flat ($\Lambda = 0$) to (A)dS ($\Lambda \neq 0$):

- 1 Lightcone gauge for Self-Dual General Relativity [2303.17866]
(Flat result: Plebanski [1975](#))
- 2 Cubic vertices in lightcone formalism for Higher-Spin Gravity
[2606.17380, 2510.22532]
(Flat result: Bengtsson-Bengtsson-Brink [1983](#))
- 3 No higher-order vertices in the chiral theory [2606.17380, 2510.22532]
(Flat result: Metsaev [1991](#))

Lightcone cubic vertices for all helicities

- Some vertices are very easy: multiply 3 gauge-invariant curvatures.
- Yang-Mills and GR vertices are not like this. Need gauge potentials.
- But – if we write the easy vertices for **all spins**
→ can “analytically continue” to the difficult ones!
- Can use higher-spin theory to “discover” Yang-Mills and GR interactions!

The easy vertices

- Consider right-handed spin- s field strengths $C_{\dot{\alpha}_1 \dot{\alpha}_2 \dots \dot{\alpha}_{2s}}^{(s)}$

- Linear field equation:

$$\partial_{\alpha}{}^{\dot{\alpha}_1} C_{\dot{\alpha}_1 \dot{\alpha}_2 \dots \dot{\alpha}_{2s}}^{(s)} = 0$$

- Lightcone ansatz for solution:

$$C_{\dot{\alpha}_1 \dots \dot{\alpha}_{2s}}^{(s)} = (q^{\alpha_1} \partial_{\alpha_1 \dot{\alpha}_1}) \dots (q^{\alpha_{2s}} \partial_{\alpha_{2s} \dot{\alpha}_{2s}}) \phi_{+s} ; \quad \square \phi_{+s} = 0 .$$

- $\implies$ Free-field Lagrangian: $\mathcal{L}_{\text{free}} = \phi_{-s} \square \phi_{+s}$.

- Cubic vertices:

$$C_{\dot{\alpha} \dots \dot{\alpha}}^{(s_1)} C_{\dot{\beta} \dots \dot{\beta}}^{(s_2)} C_{\dot{\gamma} \dots \dot{\gamma}}^{(s_3)} (\epsilon^{\dot{\alpha} \dot{\beta}})^{s_1+s_2-s_3} (\epsilon^{\dot{\beta} \dot{\gamma}})^{s_2+s_3-s_1} (\epsilon^{\dot{\gamma} \dot{\alpha}})^{s_3+s_1-s_2}$$

- These are the vertices for positive helicities $h_i \geq 0$ that satisfy triangle inequalities $h_1 + h_2 \geq h_3$ etc.

All vertices

- Write the curvature-cubed vertices in terms of the lightcone fields ϕ_h :

$$V_{h_1 h_2 h_3} = \mathbb{P}_{12}^{h_1+h_2-h_3} \mathbb{P}_{23}^{h_2+h_3-h_1} \mathbb{P}_{31}^{h_3+h_1-h_2} \phi_{h_1} \phi_{h_2} \phi_{h_3}$$

$$\text{where: } \mathbb{P}_{ij} \equiv q^\alpha q^\beta \partial_\alpha^{(i) \dot{\alpha}} \partial_{\beta \dot{\alpha}}^{(j)}$$

- So far, this was flat space. But the linearized curvatures $C_{\dot{\alpha}_1 \dots \dot{\alpha}_{2s}}^{(s)}$ are conformal! Thus, in dS, we simply multiply by the conformal factor t :

$$V_{h_1 h_2 h_3} = t^{h_{\text{tot}}-1} \mathbb{P}_{12}^{h_1+h_2-h_3} \mathbb{P}_{23}^{h_2+h_3-h_1} \mathbb{P}_{31}^{h_3+h_1-h_2} \phi_{h_1} \phi_{h_2} \phi_{h_3} \quad (1)$$

- Now, [believe eq. \(1\) for all helicities](#). This correctly reproduces the vertices of Self-Dual Yang-Mills/GR!
- For Chiral Higher-Spin Gravity, sum all vertices with an h_{tot} -dependent coupling:

$$V_{[3]} = \sum_{h_{\text{tot}} > 0} \frac{t^{h_{\text{tot}}-1}}{\Gamma(h_{\text{tot}})} \mathbb{P}_{12}^{h_1+h_2-h_3} \mathbb{P}_{23}^{h_2+h_3-h_1} \mathbb{P}_{31}^{h_3+h_1-h_2} \phi_{h_1} \phi_{h_2} \phi_{h_3}$$

Restoring locality

- Outside the original inequalities $h_1 + h_2 \geq h_3$ etc., our vertices are non-local (negative powers of derivatives).
- Can **integrate by parts**, removing all derivatives from ϕ_{h_3} .
- This is just a variational derivative $\rightarrow$ **equation of motion**.
- Procedure:

$$\mathbb{P}_{23} \rightarrow \mathbb{P}_{12} - \beta_2 \partial_t; \quad \mathbb{P}_{31} \rightarrow \mathbb{P}_{12} + \beta_1 \partial_t;$$

$$\text{where: } \beta_i \equiv q^\alpha \tilde{q}^{\dot{\alpha}} \partial_{\alpha\dot{\alpha}}^{(i)}; \quad \tilde{q}_{\dot{\alpha}} \equiv q^\alpha \partial_{\alpha\dot{\alpha}} t$$

- Result:

$$V_{h_1 h_2 h_3} = \phi_{h_3} \sum_{k_1, k_2=0}^{\infty} \frac{(-1)^{k_2}}{\Gamma(h_{\text{tot}} - k_1 - k_2)} \binom{h_3 + h_2 - h_1}{k_2} \binom{h_3 + h_1 - h_2}{k_1} \\ \times t^{h_{\text{tot}} - k_1 - k_2 - 1} \beta_1^{k_1} \beta_2^{k_2} \mathbb{P}_{12}^{h_{\text{tot}} - k_1 - k_2} \phi_{h_1} \phi_{h_2},$$

- The sum cuts off, and no negative powers of derivatives appear.

How do we know these vertices are correct?

- Prove for AdS [2606.17380]
→ Analytically continue to dS [2510.22532].
- What to prove?
Correct algebra of the (non-manifest) spacetime symmetries.
- Choose preferred lightlike direction $q^\alpha \bar{q}^{\dot{\alpha}}$ as x^- , where $x^\pm \equiv x \pm t$.
- AdS algebra: P^+ , P^- , P^y , J^{+-} , J^{+y} , J^{-y} , D , K^+ , K^- , K^y
kinematical vs. dynamical (receive interacting corrections)
- Cubic correction to P^- : the vertex $V_{[3]}$.
- Compute $J^{-y} = [K^y, P^-]$.
- Check $[P^-, J^{-y}] = 0$ to cubic order.

Main results

A series of results upgraded from flat ($\Lambda = 0$) to (A)dS ($\Lambda \neq 0$):

- 1 Lightcone gauge for Self-Dual General Relativity [2303.17866]
(Flat result: Plebanski [1975](#))
- 2 Cubic vertices in lightcone formalism for Higher-Spin Gravity
[2606.17380, 2510.22532]
(Flat result: Bengtsson-Bengtsson-Brink [1983](#))
- 3 No higher-order vertices in the chiral theory [2606.17380, 2510.22532]
(Flat result: Metsaev [1991](#))

Higher-order consistency of chiral theory

- Check $[P^-, J^{-y}] = 0$ to quartic order.
- Known for the various chiral theories in flat spacetime:
 - ▶ Self-Dual Yang-Mills
 - ▶ Self-Dual GR
 - ▶ Chiral Higher-Spin Gravity (h_{tot} -dependence of the cubic couplings becomes important)
- Trick: **conformally transform the flat result** into the AdS one.
 - ▶ Not obvious: other than Yang-Mills, the interactions aren't conformal.
 - ▶ Yet natural: we constructed the cubic vertices from conformally-invariant curvatures.
- Apply conformal transformation $\exp(\mathbf{a}K^{z+iy})$ to both P^- and J^{-y} .
- Commutator preserved, since canonical commutators are those of free theory $\rightarrow$ conformal.
- The coefficient of $\mathbf{a}^{h_1+h_2+h_3+h_4-2}\phi_{h_1}\phi_{h_2}\phi_{h_3}\phi_{h_4}$ yields the AdS result.

Main results

A series of results upgraded from flat ($\Lambda = 0$) to (A)dS ($\Lambda \neq 0$):

- 1 Lightcone gauge for Self-Dual General Relativity [2303.17866]
(Flat result: Plebanski [1975](#))
- 2 Cubic vertices in lightcone formalism for Higher-Spin Gravity
[2606.17380, 2510.22532]
(Flat result: Bengtsson-Bengtsson-Brink [1983](#))
- 3 No higher-order vertices in the chiral theory [2606.17380, 2510.22532]
(Flat result: Metsaev [1991](#))

Outlook

- Extend to fermions, supersymmetry.
- Extend the (manifest portion of) spacetime symmetry to higher-spin symmetry.
- Use the cubic-exact formulation of Chiral Higher-Spin Gravity to look for a holographic dual.
- Original goal: use cubic lightcone vertices of Higher-Spin Gravity to compute evolution between de Sitter horizons.
- Revisit the quartic order in full (non-chiral) theory. Non-localities in flat vs. AdS vs. dS?