SPECIAL RELATIVITY MIDTERM EXAM

Exercise 1. *A relativistic particle of mass m and charge q is moving with initial velocity $\mathbf{v_0} = (v_\parallel, v_\perp, 0)$. Find the particle's trajectory $x^\mu(\tau)$, where τ is proper time, under the influence of the following electromagnetic fields:*

1. $\mathbf{E} = (E, 0, 0)$ *and* $\mathbf{B} = 0$.
2. $\mathbf{E} = 0$ *and* $\mathbf{B} = (B, 0, 0)$.

Exercise 2. *The 4d Lorentz group $SO(3,1)$ is secretly the same as 3d rotation group with complex components $SO(3;\mathbb{C})$. Let us study this equivalence, by comparing the transformations of the 4d Lorentz tensor $F^{\mu\nu}$ and the 3d complex vector $\mathbf{E} + i\mathbf{B}$.*

1. *Find the real and imaginary parts of the Euclidean length-squared $(\mathbf{E} + i\mathbf{B})^2$. Express these as Lorentz scalars constructed from $F^{\mu\nu}$ using 4d tensor algebra.*
2. *Write down the Lorentz transformation matrix $\Lambda^\mu{}_\nu$ for a boost in the x direction by a boost angle θ. Write down the corresponding transformation for an antisymmetric Lorentz tensor $F^{\mu\nu}$, component-by-component. Express the same transformation as a rotation of $\mathbf{E} + i\mathbf{B}$ by an imaginary angle $i\theta$.*

Exercise 3. *It follows that by an appropriate Lorentz transformation, one can always make $\mathbf{E} + i\mathbf{B}$ point along the x axis, so that $\mathbf{E} = (E, 0, 0)$ and $\mathbf{B} = (B, 0, 0)$. Repeat Exercise 1 for this general case.*