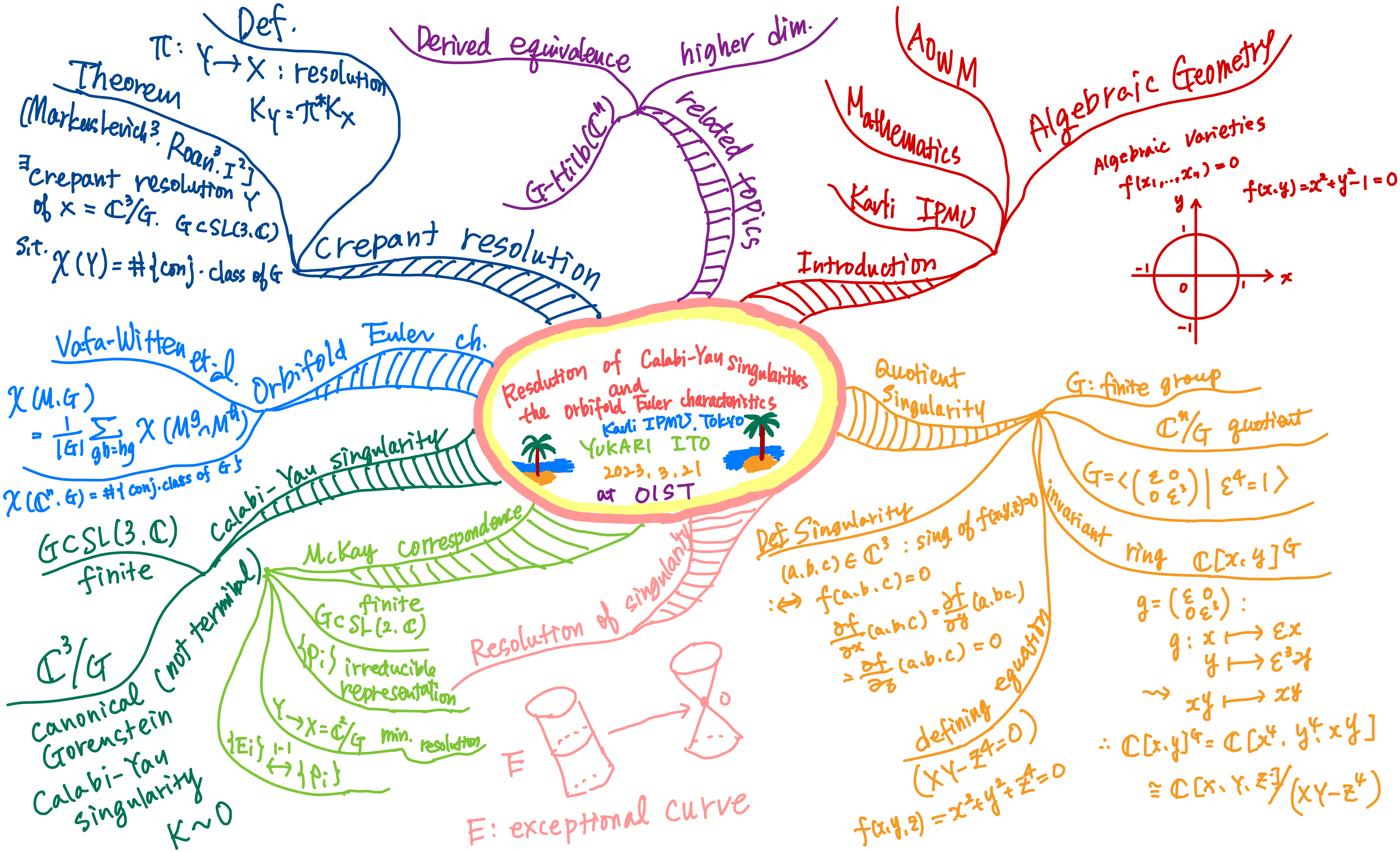

Resolution of Calabi-Yau singularities and the orbifold Euler characteristics
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at OIST
Introduction
Kavli IPMU
Mathematics
AOWM
Algebraic Geometry
Algebraic varieties
$f(x_1, \ldots, x_n) = 0$
$f(x,y) = x^2 + y^2 - 1 = 0$
y
x
1
-1
0
Quotient Singularity
G: finite group
$\mathbb{C}^n/G$ quotient
$G = \langle \begin{pmatrix} \varepsilon & 0 \\ 0 & \varepsilon^3 \end{pmatrix} \mid \varepsilon^4 = 1 \rangle$
invariant ring $\mathbb{C}[x,y]^G$
$g = \begin{pmatrix} \varepsilon & 0 \\ 0 & \varepsilon^3 \end{pmatrix}$ :
$g: x \mapsto \varepsilon x$
$y \mapsto \varepsilon^3 y$
$\rightsquigarrow xy \mapsto xy$
$\therefore \mathbb{C}[x,y]^G = \mathbb{C}[x^4, y^4, xy]$
$\cong \mathbb{C}[X,Y,Z]/(XY - Z^4)$
Def Singularity
$(a,b,c) \in \mathbb{C}^3$ : sing. of $f(x,y,z) = 0$
$:\Leftrightarrow f(a,b,c) = 0$
$\frac{\partial f}{\partial x}(a,b,c) = \frac{\partial f}{\partial y}(a,b,c) = \frac{\partial f}{\partial z}(a,b,c) = 0$
defining equation
$(XY - Z^4 = 0)$
$f(x,y,z) = x^2 + y^2 + z^4 = 0$
Resolution of singularity
E: exceptional curve
E
0
McKay correspondence
finite $G \subset SL(2,\mathbb{C})$
$\{\rho_i\}$ irreducible representation
$Y \to X = \mathbb{C}^2/G$ min. resolution
$\{E_i\} \overset{1-1}{\leftrightarrow} \{\rho_i\}$
Calabi-Yau singularity
$G \subset SL(3,\mathbb{C})$ finite
$\mathbb{C}^3/G$
Canonical (not terminal)
Gorenstein
Calabi-Yau singularity
$K \sim 0$
Orbifold Euler ch.
Vafa-Witten et al.
$\chi(M,G) = \frac{1}{|G|} \sum_{gh=hg} \chi(M^g \cap M^h)$
$\chi(\mathbb{C}^n, G) = \#\{\text{conj. class of } G\}$
Crepant resolution
Def.
$\pi: Y \to X$ : resolution
$K_Y = \pi^* K_X$
Theorem
(Markushevich[3], Roan[3], I[2])
$\exists$ crepant resolution Y of $X = \mathbb{C}^3/G$, $G \subset SL(3,\mathbb{C})$
s.t. $\chi(Y) = \#\{\text{conj. class of } G\}$
related topics
G-Hilb$(\mathbb{C}^n)$
Derived equivalence
higher dim.