

Science and Technology Group

Annual Report FY2025

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1 Introduction

My research is focused on proof theory, a subfield of mathematical logic. I investigate ordinal representation systems, which are the main tools in the ordinal analysis of mathematical theories, and which play an essential role in gauging the proof strength of axiom systems or mathematical theorems the comparative strength of which might be of interest. Besides contributing to the deeper understanding of classical ordinal representation systems, my main area of research is the investigation of representation systems derived from so-called elementary patterns of resemblance, which are of interest as they present combinatorial structures the properties of which give rise to mathematical theorems of great proof strength. I am currently also developing a new approach to the generalization of Goodstein principles.

2 Activities and Findings

During my third year as an STA, I presented my work at conferences *Logic Colloquium 2025* in Vienna, *Computability in Europe 2025* in Lisbon, and *50 Years of Explicit Mathematics* in Tübingen (online). I published a largely extended and conceptionally generalized version of my work on fundamental sequences with Bachmann property, which is now available under Open Access at *Annals of Pure and Applied Logic*. I am continuing the collaboration on Goodstein principles that was motivated and enabled by my previous results on fundamental sequences, see below. Moreover, I am elaborating an independent new approach to the generalization of Goodstein principles (preliminary version on the arXiv server) that I discovered recently and from which I expect to find connections to patterns of resemblance via ordinal arithmetic.

3 Collaborations

A collaboration with researchers from Ghent University (Belgium) and the University of Barcelona (Spain) on Goodstein principles of higher complexity and ordinal notation systems in general is ongoing, with the aim to apply my results on fundamental sequences to the theory of Goodstein principles, which recently advanced significantly by the work of my collaborators. This ongoing collaboration is expected to benefit crucially from my recent independent approach to Goodstein principles.

4 Publications and other output

G. Wilken: *Fundamental sequences based on localization*. *Annals of Pure and Applied Logic* 177 (2026), pp. 1-43, Elsevier.

G. Wilken: *Goodstein principles: recent developments*. Talk at: *Logic Colloquium 2025*, TU Vienna, on July 10th, 2025.

G. Wilken: *Fundamental sequences and Goodstein principles*. Talk at: *Computability in Europe 2025*, University of Lisbon, on July 16th, 2025.

G. Wilken: *Fundamental sequences and Goodstein principles*. Invited online talk at: *50 Years of Explicit Mathematics*, University of Tübingen, on September 20th, 2025.