Dynamic inverse problems regularized with Wasserstein-1 transport

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The (classical, balanced) *Wasserstein-p distance* can be used as a measure of how close a source and sink mass distribution (with equal mass) are. In recent years, the Wasserstein-2 distance has been employed in the temporal regularization of *dynamic inverse problems*. The so-called *Benamou–Brenier formula* states that it can be written as the square root of the performed physical work through the transport from the source to the sink. In my talk, I will instead focus on dynamic inverse problems regularized with Wasserstein-1 transport. The Wasserstein-1 distance can be interpreted as the optimal transport cost with respect to the Euclidean distance: it equals $\inf_\pi \int |x-y| \mathrm{d}\pi(x,y)$, where measure element $\mathrm{d}\pi(x,y)$ indicates the (infinitesimal) amount of mass moving from location x to y. I will explain a novel dynamic inverse problem on time-parameterized curves in the induced *Wasserstein*-1 *(metric) space*. It is a natural extension of static sparse optimization problems such as lasso or TV regularization. One essential difference to classical regularization with Wasserstein-2 transport is that it allows for discontinuous decision variables (realized as BV curves). Despite this weak regularity requirement and the non-differentiability of the cost function $(x,y) \mapsto |x-y|$, it is possible to prove the existence of a sparse solution and its characterization. I will present this result. Further, I will detail an adaption of the fully-corrective generalized conditional gradient method to the problem and highlight a natural discretization approach. Finally, I will show some numerical examples.

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