GENERAL RELATIVITY HOMEWORK – WEEK 4

Exercise 1. *Consider the Lagrangian of a charged scalar field coupled to electromagnetism:*

$$L[\Phi, A_\mu] = -D_\mu\bar{\Phi}D^\mu\Phi - m^2\bar{\Phi}\Phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \ , \tag{1}$$

where:

$$D_\mu\Phi \equiv (\partial_\mu - iqA_\mu)\Phi \ ; \quad D_\mu\bar{\Phi} \equiv (\partial_\mu + iqA_\mu)\bar{\Phi} \ ; \quad F_{\mu\nu} \equiv 2\partial_{[\mu}A_{\nu]} \ . \tag{2}$$

Derive from this Lagrangian the field equations for Φ *and for* A_μ*. What is the current* J^μ *that appears in the field equation for* A_μ*?*

Exercise 2. *The Lie derivative along a vector field* v^μ *is defined on scalars and vectors as:*

$$\mathcal{L}_v f = v^\mu\partial_\mu f \ ; \quad \mathcal{L}_v w^\mu = v^\nu\partial_\nu w^\mu - w^\nu\partial_\nu v^\mu \ . \tag{3}$$

Using these and the Leibnitz rule for the derivatives of products, work out the formulas for $\mathcal{L}_v u_\mu$ *(Lie derivative of a covector) and* $\mathcal{L}_v g_{\mu\nu}$ *(Lie derivative of a rank-2 tensor with lower indices).*