GENERAL RELATIVITY HOMEWORK – WEEK 3

Exercise 1. *Let's play with the stress-energy tensor of a fluid.*

1. *Consider a fluid of free particles, moving all with the same velocity* $\mathbf{v}$*, with energy density* ρ*. Write down all the components* T^{tt}, T^{ti}, T^{ij} *of the fluid's stress-energy tensor* $T^{\mu\nu}$*. Find the trace* T^{μ}_{μ}*.*

2. *Same, but this time the particles are moving randomly in all directions, still with the same magnitude* $|\mathbf{v}|$ *of the velocity.*

3. *What happens to* T^{μ}_{μ} *in the speed-of-light case* $|\mathbf{v}| = 1$*?*

Exercise 2. *Now let's play with the stress-energy tensor of the electromagnetic field.*

1. *Let's construct all possible symmetric matrices* $A_{\mu\nu} = A_{\nu\mu}$ *out of the electromagnetic field strength* $F_{\mu\nu}$ *and the Minkowski metric* $\eta_{\mu\nu}$*. There are exactly two such matrices quadratic in* $F_{\mu\nu}$*. Find them.*

2. *The energy density of the electromagnetic field is* $\rho = \frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2)$*. Using the answer to part 1, deduce the Lorentz-covariant expression for the complete stress-energy tensor* $T^{\mu\nu}$ *of the electromagnetic field.*

3. *What is the trace* T^{μ}_{μ}*?*