Experiments on photonic qudits

Jacquiline (Jacqui, Jacq) Romero

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Plan…

Review photonic qudits, focus on transverse mode (shape)

Convince you that ignorance of the whole does not imply ignorance of the parts.

Convince you that self-guided tomography is a robust and efficient way to do quantum state tomography.

Look at a method to implement high-d gates in shape.

Review some photonic entanglement experiments.

Why photons?

The world is cold at optical frequencies…

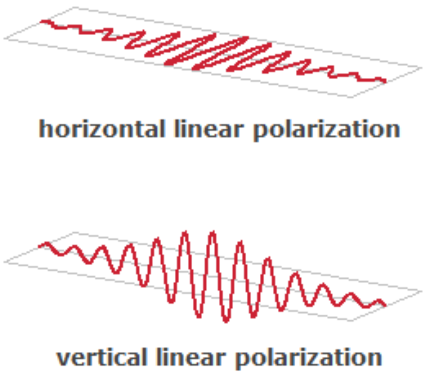
The frequency of photonic excitations is $\omega \sim 10^{14}$

At room temperature: $\dfrac{\hbar\omega}{k_B T} \sim 10$

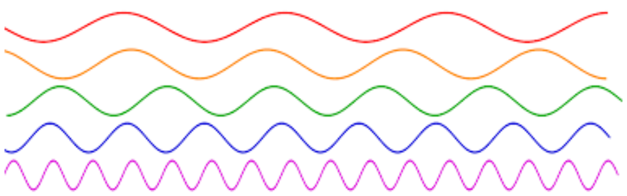
The average number of background thermal photons is: $\bar{n}(\omega) = \dfrac{1}{e^{\frac{\hbar\omega}{k_B T}} - 1} \sim 0$

Quantum effects manifest at room temperature.

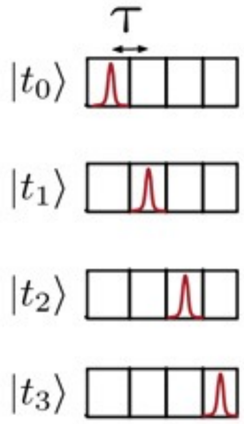
Photonic qudits



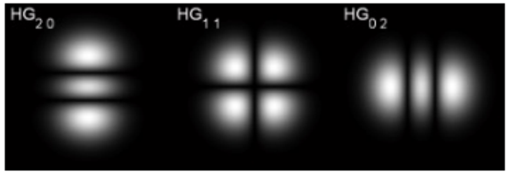
polarisation

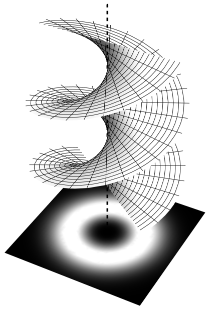


wavelength/frequency



time bins





transverse mode/shape
(OAM/twisted light)

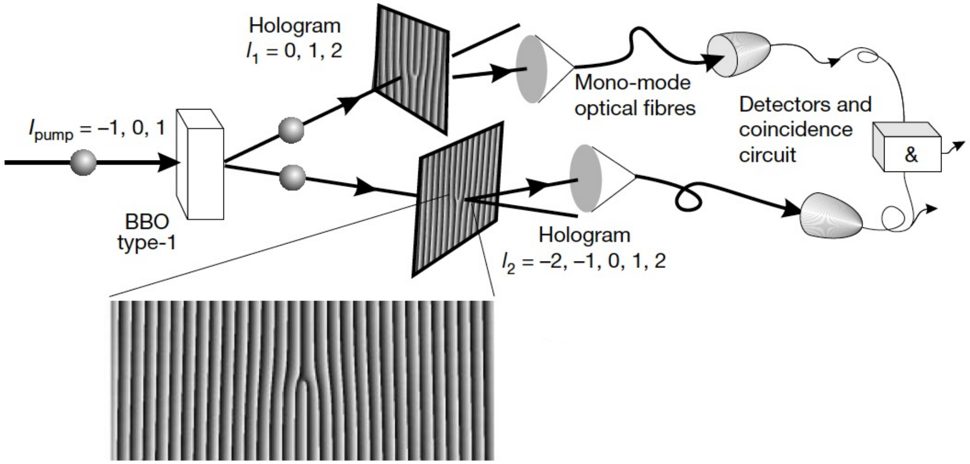
rp-photonics.com

Entanglement of photon orbital angular momentum (OAM)

Entanglement is the quintessential quantum correlation.

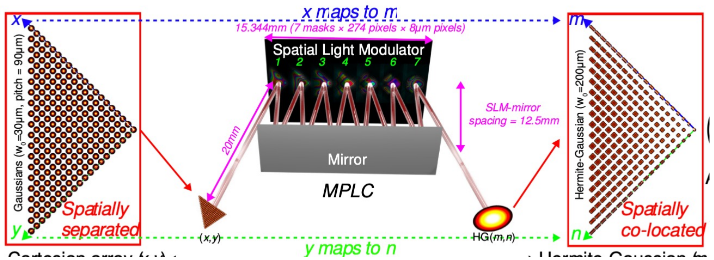
Entanglement of OAM has been demonstrated.

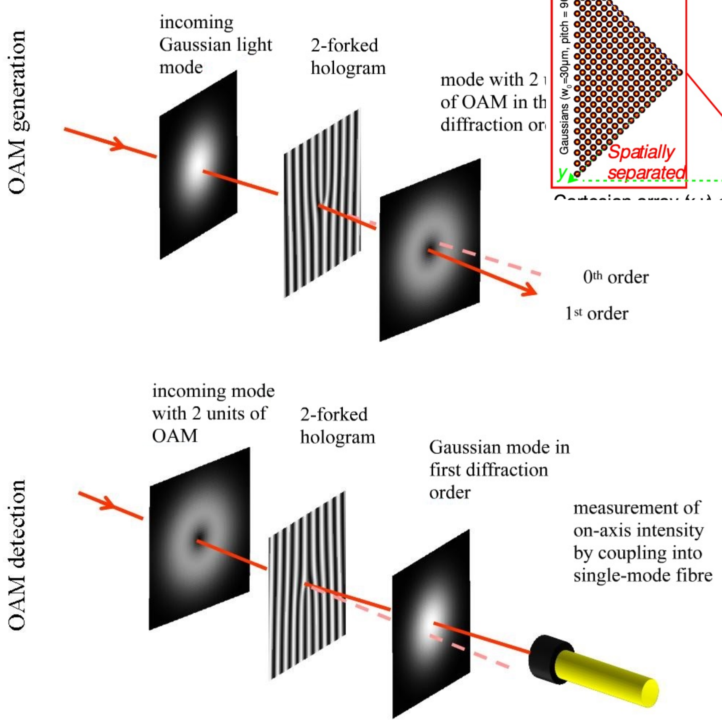
Mair et al, Nature 412, 313 (2001)

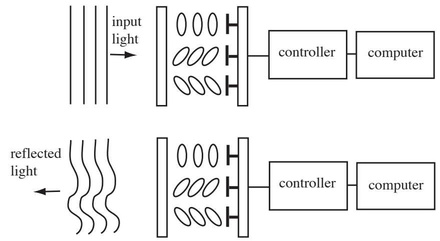


One technological step…

Fontaine et al.







Use a spatial light modulator (SLM) instead of fixed holograms to improve versatility.

It's not just twist. Photons have many shapes.

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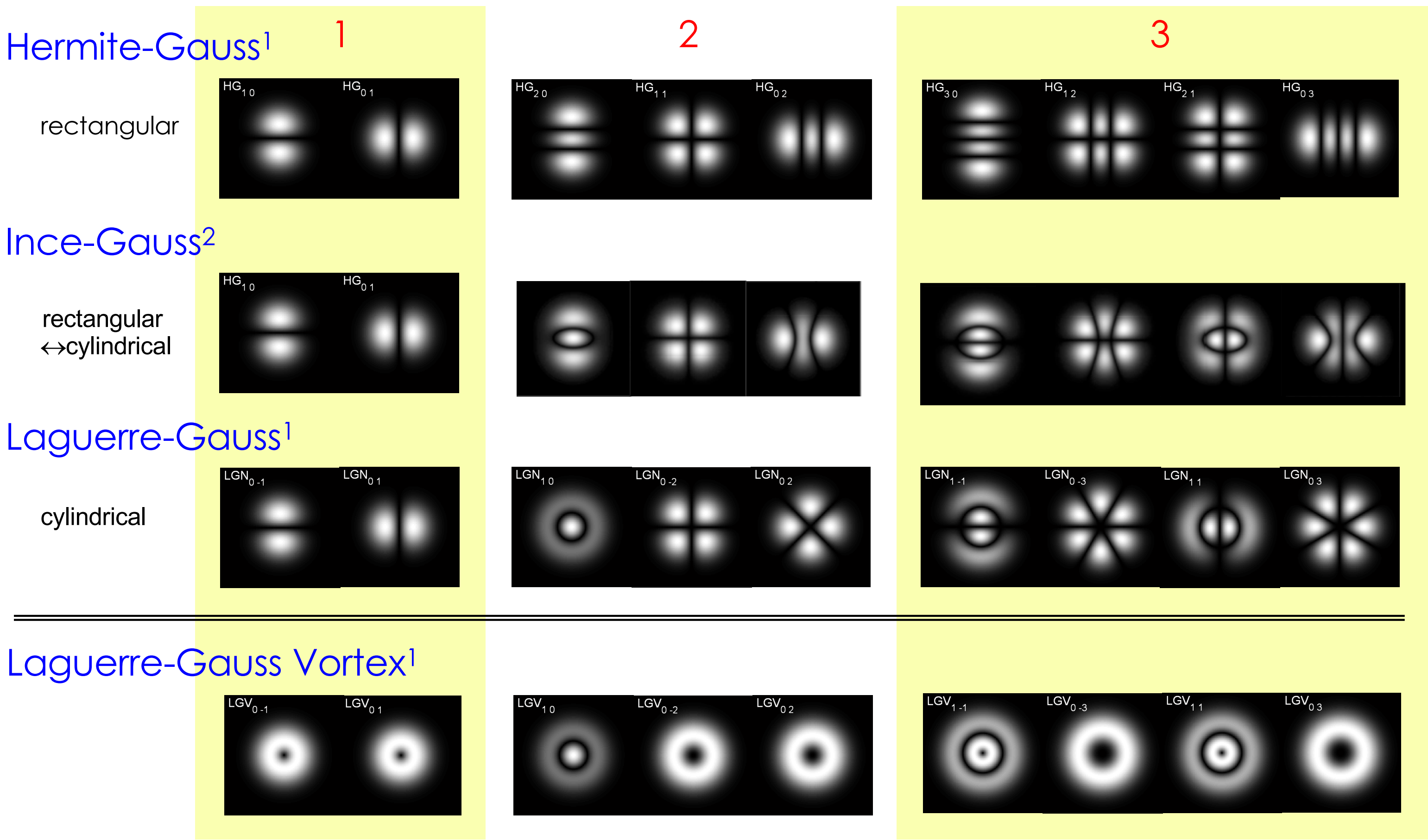
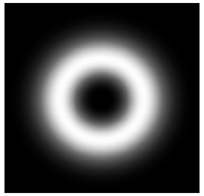
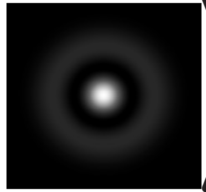


1. Siegman, Lasers (1986)

2. Bandres *et al.*, Optics Letters 29, 144 (2004)

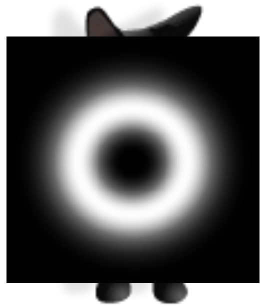
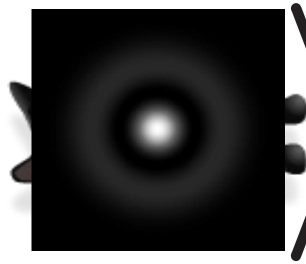
Quantum states from the shape of light

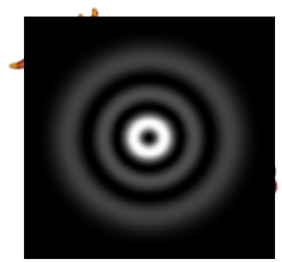
$|\psi\rangle = \alpha \, |$  $\rangle + \beta \, |$  $\rangle$

$|$  $\rangle = \alpha \, |$  $\rangle + \beta \, |$  $\rangle$

qubits

Quantum states from the shape of light

$\left| \text{ZOO} \right\rangle = \alpha \left|$  $\right\rangle + \beta \left|$  $\right\rangle$

$+ \gamma \left|$  $\right\rangle + \eta \left|$  $\right\rangle$

Qudits@UQ

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Convince you that ignorance of the whole does not imply ignorance of the parts.

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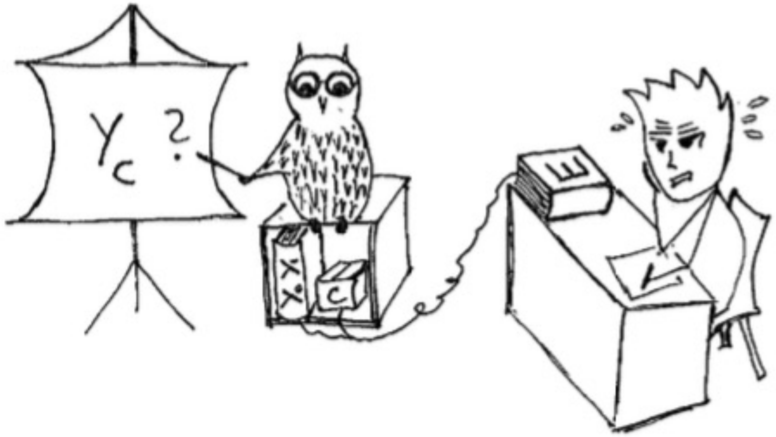
Does ignorance of the whole imply ignorance of the parts?

Michael Kewming

Sally Shrapnel

Andrew White



Vidick and Wehner, Phys Rev Lett 107, 030402 (2011)

For the PhD students…mostly…

I miss a punch line, or a new insight/surprise…that would give a real convincing case to support publication in a high-impact journal such as PRL. The mathematical/quantum physical concept has been presented in papers before. The experimental technique didn't require any improvements over state-of-the-art techniques or push any limits…The authors don't provide a convincing case of what to do …now with the developed techniques.

The authors answered my questions, and I read the article again in detail. There was one additional part of the work that I find quite interesting and actually unique.

The authors give a very nice real-world interpretation of the statement, which comes from pure probability theory and contextuality considerations, which are usually not particularly, accessible to non-experts.

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PHYSICAL REVIEW LETTERS 124, 250401 (2020)

Hiding Ignorance Using High Dimensions

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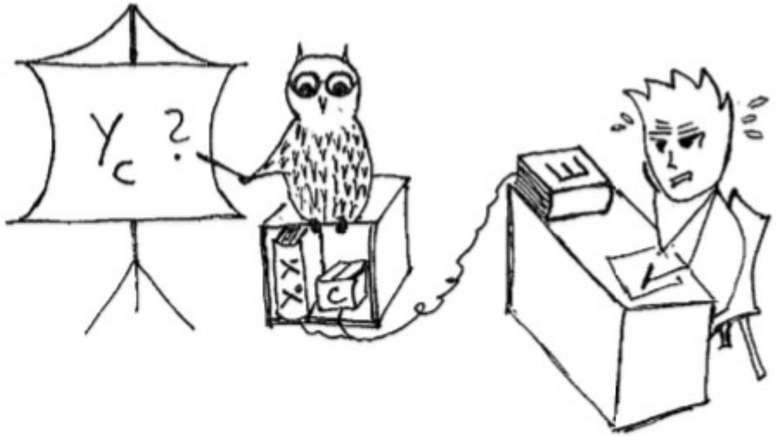
(Received 1 July 2019; revised manuscript received 17 November 2019; accepted 20 May 2020; published 22 June 2020)

The absence of information—entirely or partly—is called ignorance. Naturally, one might ask if some ignorance of a whole system will imply some ignorance of its parts. Our classical intuition tells us yes, however quantum theory tells us no: it is po
despite some ignorance of the whole, it is
S. Wehner, Phys. Rev. Lett. 107, 030402 (2
requires controlling and measuring quantum
experimental evidence using the transverse sp
dimensional quantum phenomena.

DOI: 10.1103/PhysRevLett.124.250401

Does ignorance of the whole imply ignorance of the parts?



Vidick and Wehner, Phys Rev Lett 107, 030402 (2011)

Wholes and parts

Entangled pair

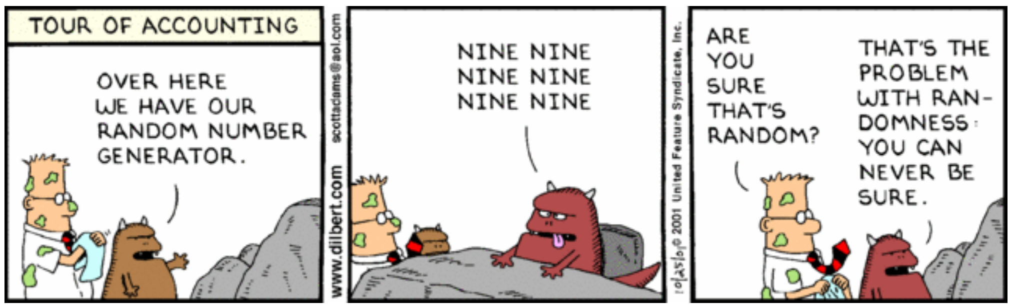
Good old *dit* string

Dit string

Dit for *d* "letters"

$$Y = \boxed{\boxed{Y_0}\boxed{Y_1}}$$

Randomness increases as d increases.



Quantifying ignorance: min-entropy

$p_{\text{guess}}(Y|E)$ is the highest success probability of guessing Y given encoding strategy E

Min-entropy: $$H_\infty(Y|E) = -\log p_{\text{guess}}(Y|E)$$

Min-entropy is large when $p_{\text{guess}}(Y|E)$ is small.

Min-entropy is large when ignorance is large .

Theorem (Vidick-Wehner min-entropy splitting inequality)

For any random variable $Y=Y_0Y_1$ and encoding E, **there exists** a random variable $C \in \{0,1\}$ such that

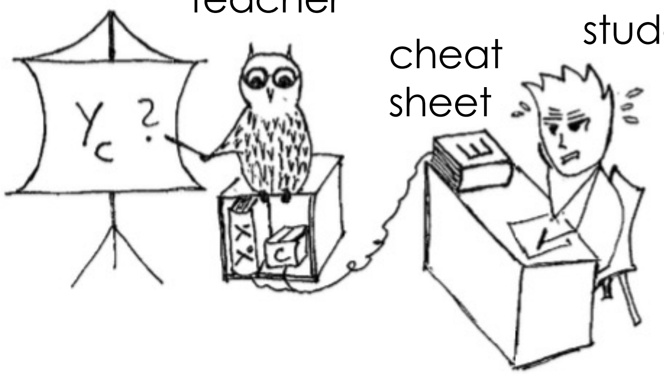
$$\underbrace{H_\infty(Y_C|E,C)}_{\text{part}} \geq \frac{\overbrace{H_\infty(Y|E)}^{\text{whole}}}{2}.$$

The ignorance of a part is at least half of the ignorance of the whole.

There exists a pointer $C \in \{0,1\}$ that points to the part where the student has large ignorance (the unknown part).

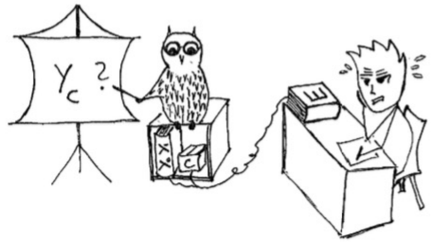
There is always a test that reveals which part of the dit string the student is ignorant of.

Teacher's task: find C that points to the part where there is large ignorance.

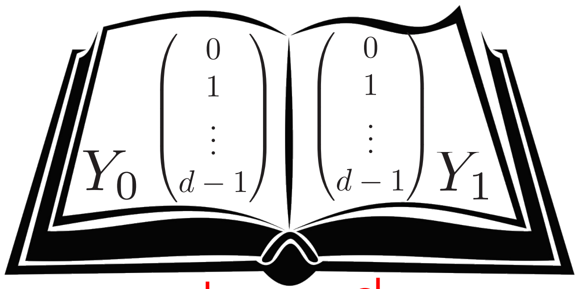


Vidick and Wehner, Phys. Rev. Lett. 107, 030402 (2011)

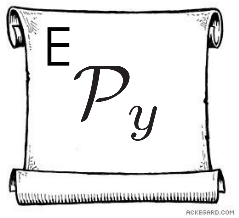
Theorem in terms of probability



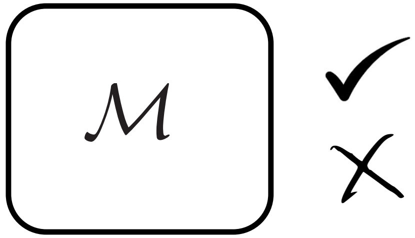
$$Y = Y_0 \times Y_1$$
$$y = y_0 y_1$$



d d



d



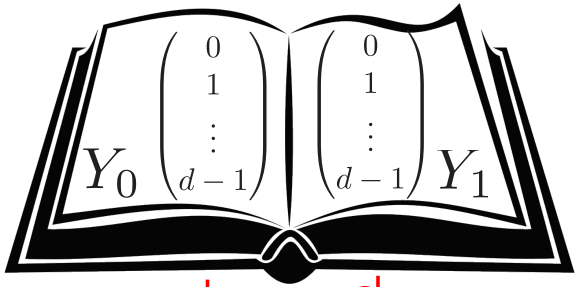
Probability of guessing Y_C given encoding E is given by: $p_{guess}(Y_c|E,C)$

There exists a pointer C ∊ {0,1} such that $\dfrac{1}{\sqrt{d}} \geq p_{guess}(Y_c|E,C)$

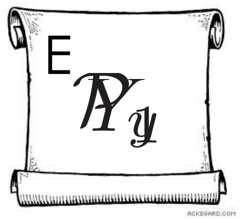
How does the teacher spot ignorance?

$$Y = Y_0 \times Y_1$$

$$y = y_0 y_1$$



d d



d

$\mathcal{M}$ ✓ ✗

$$\frac{1}{\sqrt{d}} \geq p_{guess}(Y_c|E,C)$$

$$\frac{1}{\sqrt{d}} \geq 1/d$$

The optimal classical strategy: encode one part, e.g. Y_1

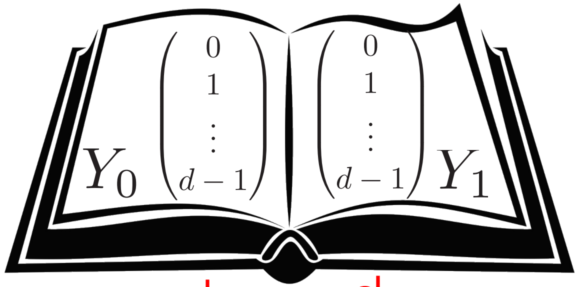
$p_{guess}(Y_1|E = Y_1) = 1$ best guessing probability

$p_{guess}(Y_0|E = Y_1) = 1/d$ worst: guessed at random

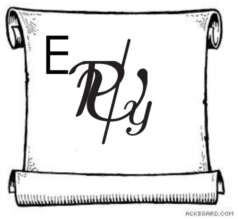
Teacher has to find which part the student is guessing at random.

Quantum Strategy

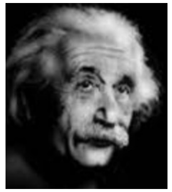
$$Y = Y_0 \times Y_1$$
$$y = y_0 y_1$$

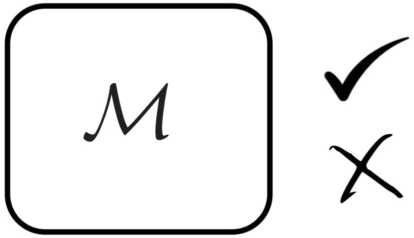


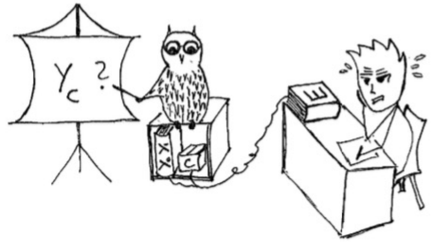
d d



d





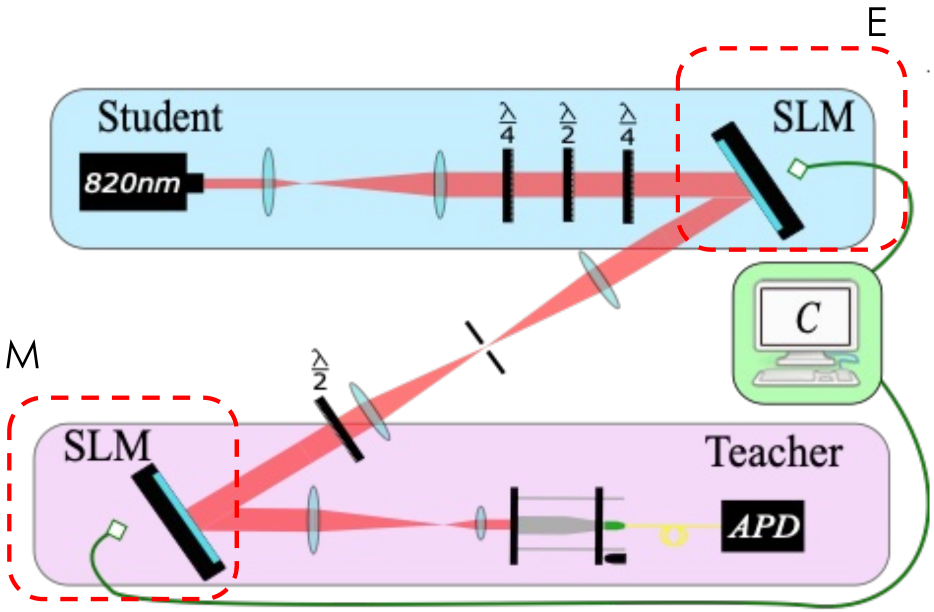


The quantum strategy is encode

$$|\Psi_y\rangle = \frac{1}{\sqrt{2\left(1+1/\sqrt{d}\right)}} X_d^{y_0} Z_d^{y_1} \left(\mathbb{I} + F\right)|0\rangle$$

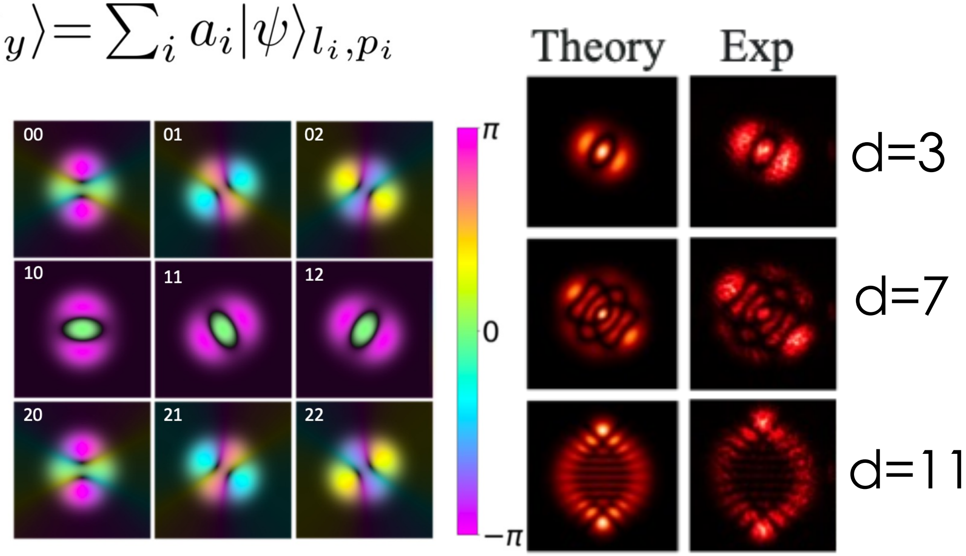
Qudit encoding for a 2-dit string

Experiment



$\langle \Phi_y |$

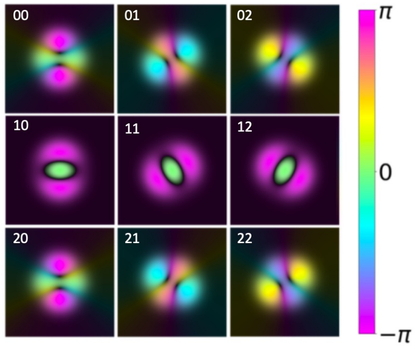
$|\Psi_y\rangle = \sum_i a_i |\psi\rangle_{l_i, p_i}$



The probabilities are obtained as:

$$\mathrm{tr}\left(\rho_y^E\, M_y\right) = \frac{|\langle \Phi_y | \Psi_y \rangle|^2}{|\langle \Psi_y | \Psi_y \rangle|^2} = \frac{O_y}{N_y}$$

Quantum strategy



$$p_{guess}(Y_0|E=\rho_y^E) = \frac{1}{2}\left(1+\frac{1}{\sqrt{d}}\right)$$

$$p_{guess}(Y_1|E=\rho_y^E) = \frac{1}{2}\left(1+\frac{1}{\sqrt{d}}\right)$$

The guessing probability for either part is now the same.

Neither of the parts is being guessed randomly (neither is 1/d).

Recall the theorem:

There exists a pointer C ∈ {0,1} such that

$$\frac{1}{\sqrt{d}} \geq p_{guess}(Y_c|E,C)$$

If the quantum encoding is used, there is no pointer C, hence the teacher cannot find the unknown part.

$$\frac{1}{\sqrt{d}} \geq \frac{1}{2}\left(1+\frac{1}{\sqrt{d}}\right) \quad \times$$

You can score higher with a quantum encoding...

Classical optimal strategy:

$$p_{guess}(Y_1|E=Y_1)=1$$

$$p_{guess}(Y_0|E=Y_1)=1/d$$

$$\left(\frac{1}{2}\times 1\right)+\left(\frac{1}{2}\times\frac{1}{d}\right)$$

d=2: 75 %

d=6: 58 %

Quantum strategy:

$$p_{guess}(Y_0|E=\rho_y^E)=\frac{1}{2}\left(1+\frac{1}{\sqrt{d}}\right)$$

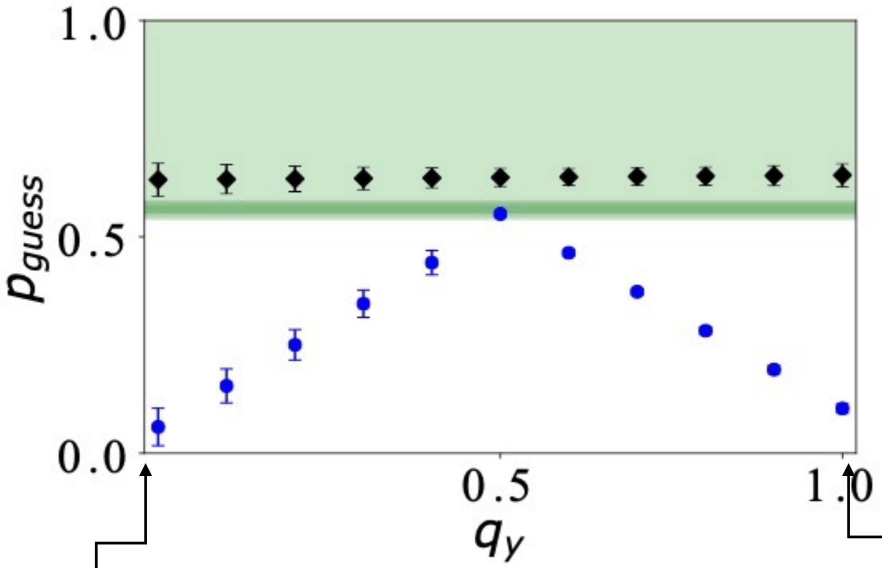
$$p_{guess}(Y_1|E=\rho_y^E)=\frac{1}{2}\left(1+\frac{1}{\sqrt{d}}\right)$$

$$\left(\frac{1}{2}\times\frac{1}{2}\left(1+\frac{1}{\sqrt{d}}\right)\right)+\left(\frac{1}{2}\times\frac{1}{2}\left(1+\frac{1}{\sqrt{d}}\right)\right)$$

d=2: 85 %

d=6: 70 %

There is no test that will reveal which part the student doesn't know…



Teacher always asks from Y_1

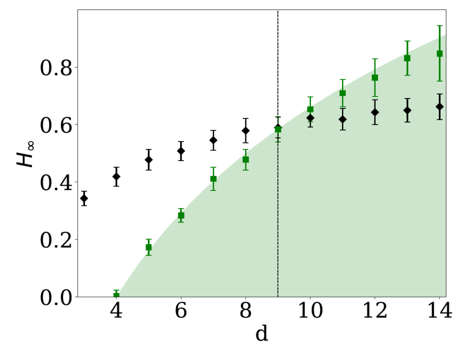
Teacher always asks from Y_0

For the quantum encoding, p_{guess} is constant (64% for d=13) — the teacher can not point to the source of ignorance.

For a classical strategy of encoding $E=Y_0$ for $q_y < 0.5$ and encoding $E=Y_1$ for $q_y > 0.5$, p_{guess} is highly correlated with C–- the teacher can point to the source of ignorance.

With a 1-qudit encoding of the 2-dit string…

The student can get a higher score.



The teacher cannot point to the part that the student is ignorant of.
The student can hide their ignorance.

Why are you not using a single-photon source, a.k.a heralded SPDC ?

Short answer: we don't have to because we are looking at single photon detection probabilities.

Gerard Milburn

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

$$|\Psi\rangle = |\alpha_1\rangle \otimes |\alpha_2\rangle \otimes \ldots \otimes |\alpha_d\rangle = |\alpha_1, \alpha_2, \ldots, \alpha_d\rangle$$

Heralded SPDC is thermal in theory, but Poissonian in practice because the heralding window is much longer than the coherence time of the photons.

Blauensteiner et al., Phys. Rev. A 79, 063846 (2009)

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The quantum regime is attained for classical coherent light fields when the time interval between photodetection events, with an ideal, perfectly efficient detector, is much greater than the transit time of radiation through the system. If the state of the radiation field produced by the laser is represented by the coherent state $|\alpha\rangle$, then the attenuated laser field is given by $|\sqrt{\eta}\alpha\rangle$, where $|\alpha|^2$ is the photon flux prior to attenuation, and $\eta|\alpha|^2$ is the photon flux following attenuation. Attenuation does not destroy the coherence of the beam, or affect the coherence time. However, a reduction of the photon flux increases the integration

Hariharan and Sanders, 1996

E. WOLF, PROGRESS IN OPTICS XXXVI

II

QUANTUM PHENOMENA IN OPTICAL INTERFEROMETRY

What if you use a strong laser instead?

Short answer: the intensity will be proportional to the single photon detection probabilities.

Steve Barnett

On single-photon and classical interference

Stephen M. Barnett
School of Physics and Astronomy, University of Glasgow, Glasgow G12 8QQ, United Kingdom
E-mail: stephen.barnett@glasgow.ac.uk

Abstract. It has often been remarked that single-photon interference experiments, however complicated, seem to behave very much in the same way as those performed in the classical regime, using the field generated by a laser. This observation has the status of being 'well-known to those who know it', but perhaps mysterious to others. We discuss the reasons underlying the similarity and also some of the limitations of this simple idea.

What is quantum about this???

$$\boxed{|\Psi_y\rangle = \frac{1}{\sqrt{2\left(1+1/\sqrt{d}\right)}} X_d^{y_0} Z_d^{y_1} \left(\mathbb{I}+F\right)|0\rangle}$$

We detected single photons in our experiment…

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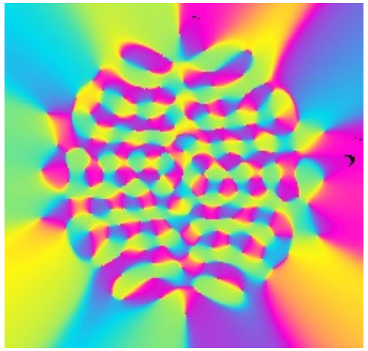
Review some photonic entanglement experiments.

Robust and efficient high-dimensional quantum state tomography



Markus Rambach

Michael Kewming

Kaumudi Goswami

Mahdi Qaryan

Zhonghua Ma

Andrew White

Chris Ferrie

The University of Queensland
Australia

PHYSICAL REVIEW LETTERS **126**, 100402 (2021)

Editors' Suggestion | Featured in Physics

Robust and Efficient High-Dimensional Quantum State Tomography

Markus Rambach,[1,2,*] Mahdi Qaryan,[1,2] Michael Kewming,[1,2] Christopher Ferrie,[3]
Andrew G. White,[1,2] and Jacquiline Romero[1,2,†]
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(Received 1 October 2020; revised 12 January 2021; accepted 5 February 2021; published 10 March 2021)

The exponential growth in Hilbert space with increasing size of a quantum system means that accurately characterizing the system becomes significantly harder with system dimension d. We show that self-guided tomography is a practical, efficient, and robust technique of measuring higher-dimensional quantum states. The achieved fidelities are over 99.9% for qutrits ($d = 3$) and ququints ($d = 5$), and 99.1% for quvigints ($d = 20$)—the highest values ever realized for qudit pure states. We also show excellent performance for mixed states, achieving average fidelities of 96.5% for qutrits. We demonstrate robustness against experimental sources of noise, both statistical and environmental. The technique is applicable to any higher-dimensional system, from a collection of qubits through to individual qudits, and any physical realization, be it photonic, superconducting, ionic, or spin.

DOI: 10.1103/PhysRevLett.126.100402

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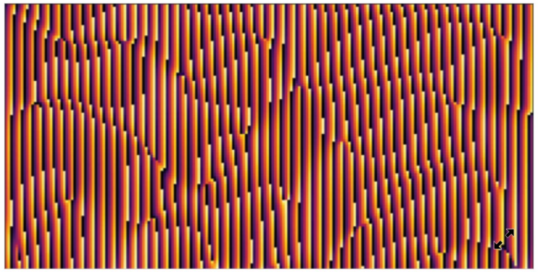
SYNOPSIS

Measuring Higher Dimensional "Qudits" for Computation

March 10, 2021 • *Physics* 14, s34

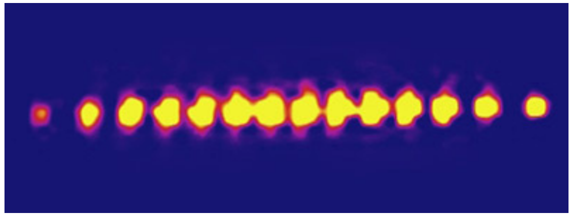
With a technique called self-guided tomography, researchers accurately measure the states of qudits—quantum systems like qubits but with more than two dimensions.

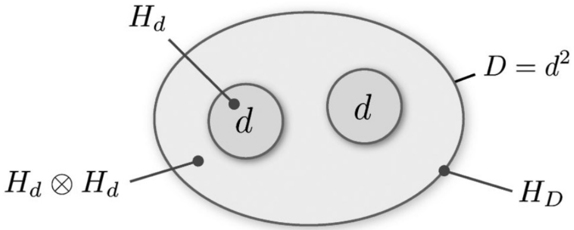


M. Rambach/University of Queensland

Quantum State Tomography

What is the quantum mechanical description of a physical system?





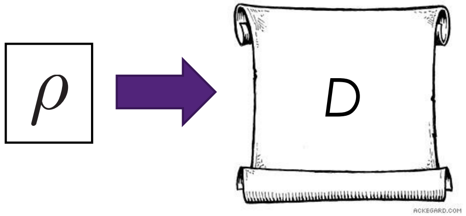
The parameter space grows as 4^N-1 for N qubits.

Why do you always use qubits?

Because any *d* can be represented by N qubits

The parameter space grows as $\mathbf{d}^{2N}$-1 for N qu**d**its.

Quantum State Tomography



There is some true state which generates data.

Data set needs to be saved.

$$\Pr(D \mid \rho) = \prod_k \mathrm{Tr}(\rho E_k)$$

From a complete set of measurements, find the quantum state.
Usually done in post-processing, e.g. $O(d^4)$ for maximum likelihood.

computationally expensive post-processing

sensitive to noise

no post-processing

efficient

robust

Self-guided Tomography (SGT)

no post-processing   efficient   robust

$|\psi\rangle$ unknown target state

From current estimate, $|\sigma_k\rangle$

Choose one random direction, $(\Delta_k)_j \in \{1, -1, i, -i\}$

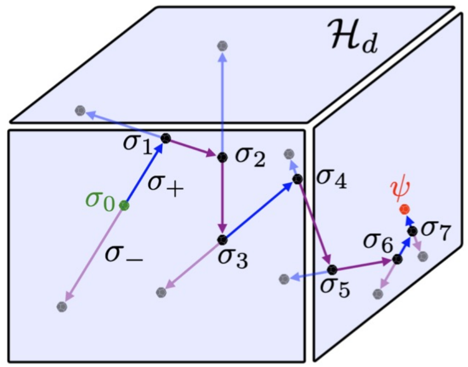
Measure the two projections, $|\sigma_\pm\rangle = |\sigma_k \pm \beta_k \Delta_k\rangle$

$\beta_k = b/(k+1)^t \quad (b, t)$ hyperparameters

Calculate the gradient $g_k = \dfrac{f(\sigma_k + \beta_k \Delta_k) - f(\sigma_k - \beta_k \Delta_k)}{2\beta_k} (\Delta_k^{-1})^*$

Update to the next estimate $|\sigma_{k+1}\rangle = |\sigma_k + \alpha_k g_k\rangle$

$\alpha_k = a/(k+1+A)^s \, (a, A, s)$ hyperparameters



$$f(\sigma) = |\langle \sigma | \psi \rangle|^2$$

distance measure that can be estimated from the experiment

Ferrie, PRL 113, 190404 (2014)

Photon shape as a qudit

$$|\psi\rangle = \sum_i c_i |l_i, p_i\rangle$$

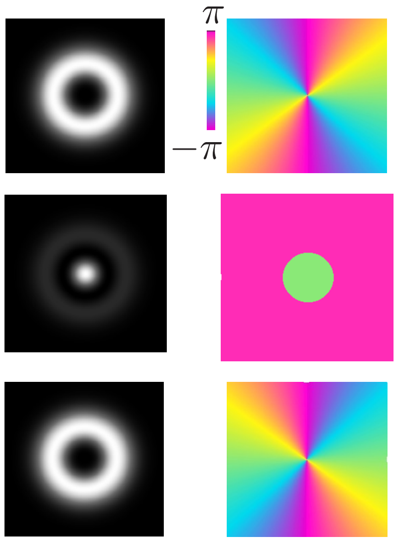
qutrit (d=3) logical basis

(l,p)

(-2,0)

(0,1)

(2,0)



ququint (d=5) logical basis

(l,p)

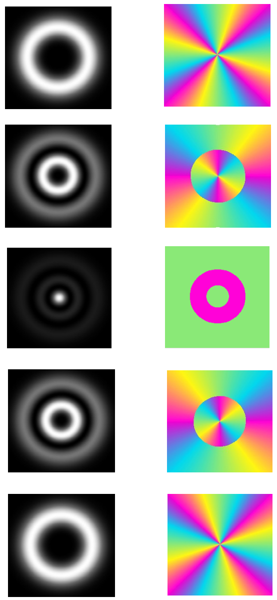
(-4,0)

(-2,1)

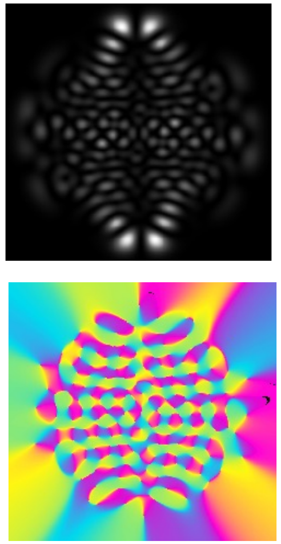
(0,2)

(2,1)

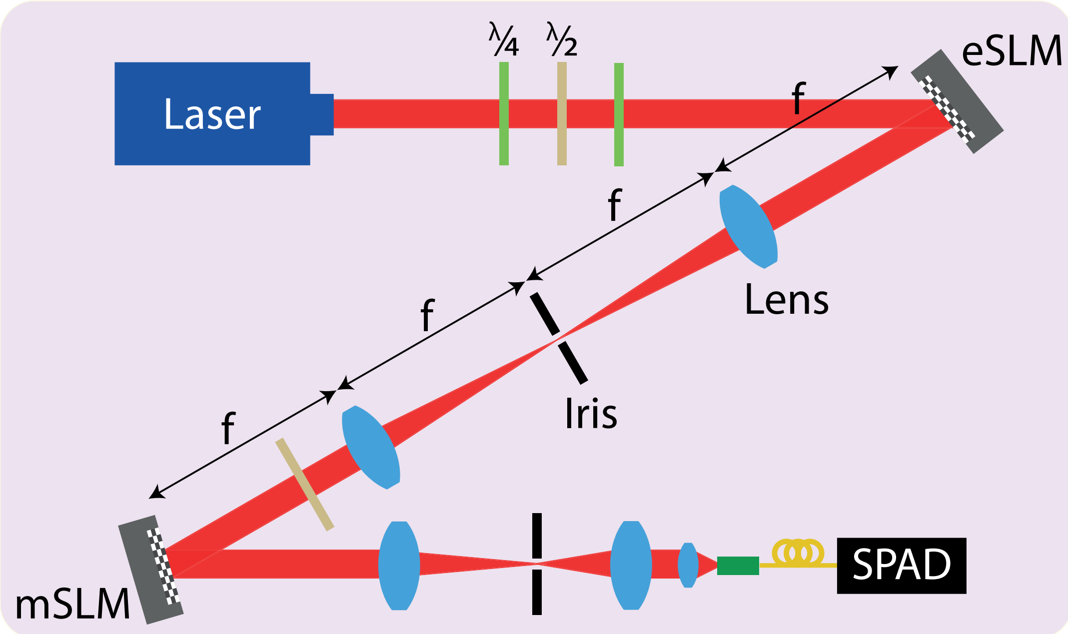
(4,0)



quvigint (d=20)



Experiment: prepare and measure



$$f(\sigma) = |\langle \sigma | \psi \rangle|^2$$

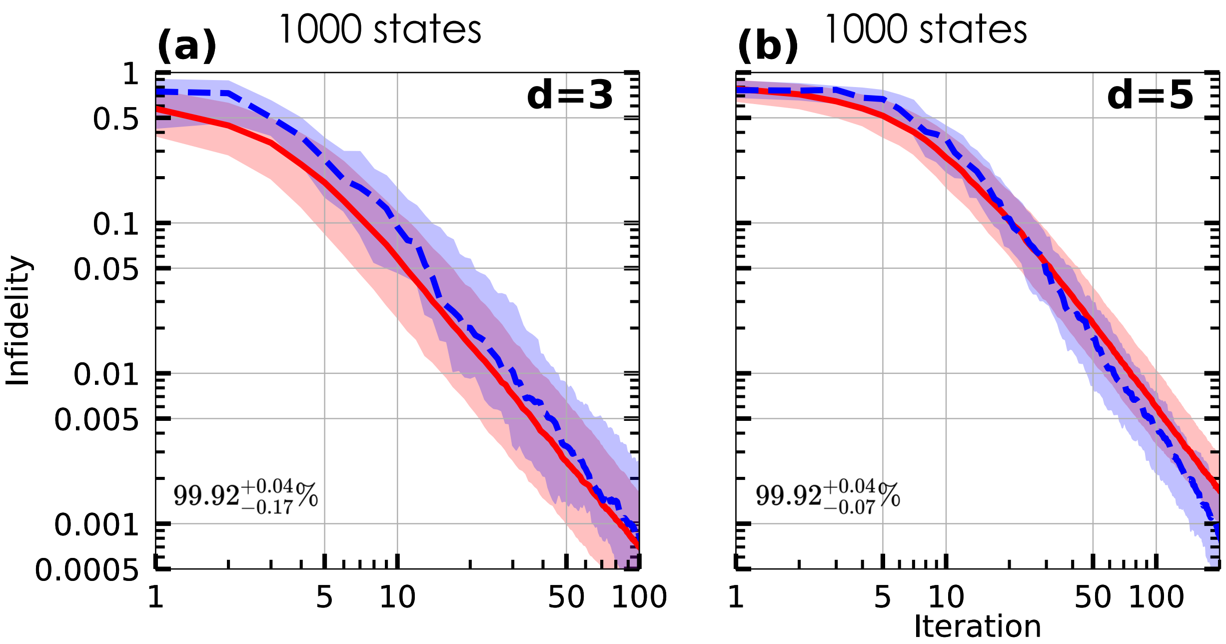
$$f(\sigma_+) - f(\sigma_-) = f(\sigma_k + \beta_k \Delta_k) - f(\sigma_k - \beta_k \Delta_k)$$

$$\frac{|\langle \sigma_k + \beta_k \Delta_k \mid \psi \rangle|^2 - |\langle \sigma_k - \beta_k \Delta_k \mid \psi \rangle|^2}{|\langle \sigma_k + \beta_k \Delta_k \mid \psi \rangle|^2 + |\langle \sigma_k - \beta_k \Delta_k \mid \psi \rangle|^2} = \frac{N_+ - N_-}{N_+ + N_-}$$

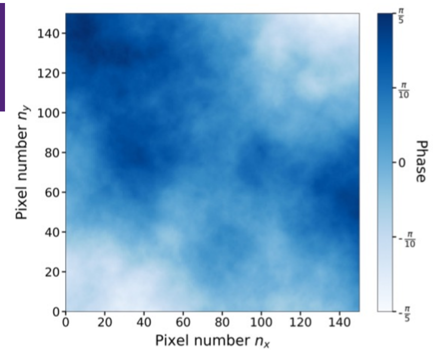
Note we are not using probabilities, just the counts.

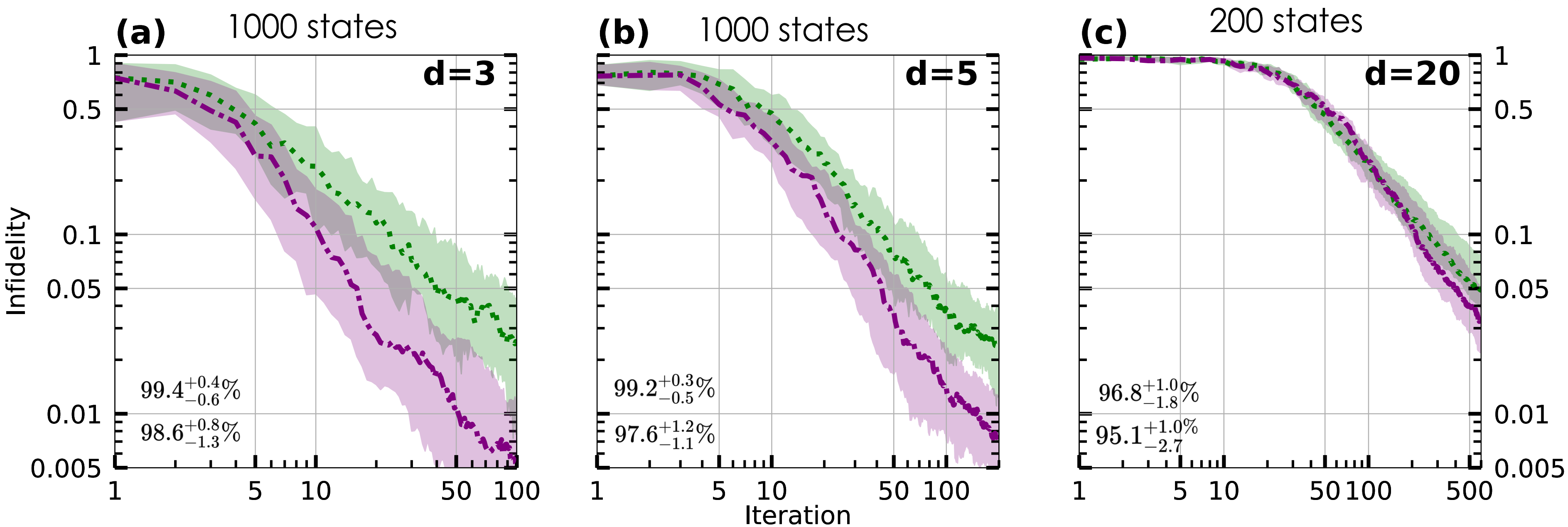
That makes it more robust against error in probabilities (e.g. due to mode-dependent efficiencies).

Results: For low statistical noise (~10^5 photons)



Results: Robustness

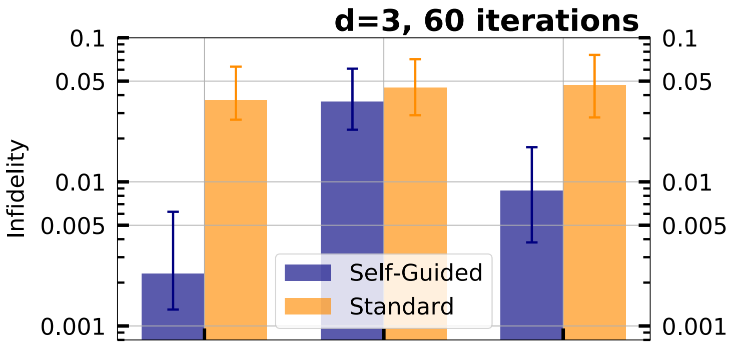




Green: high statistical noise (N~80 for qutrits and quiquints, N~1000 for quvigint)

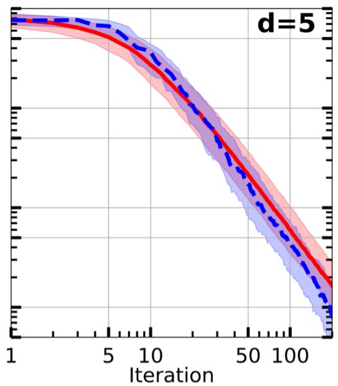
Purple: weak atmospheric turbulence ($C_n^2 = 10^{-18}\ m^{-2/3}$)

Results: Comparison with standard tomography



Self-guided tomography achieves lower infidelity than standard tomography.

Standard tomography infidelity stagnates.



Increase iterations to get smaller infidelity.

Results: Mixed state

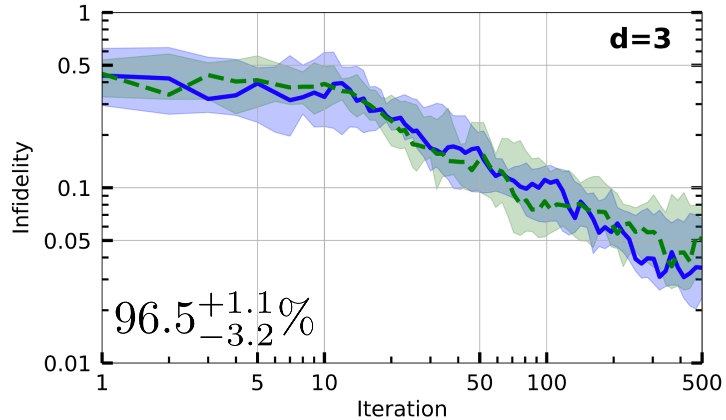
Target state with a range of purities (~0.3 to 0.8):

$$\rho = \begin{pmatrix} p_1 & 0 & 0 \\ 0 & p_2 & 0 \\ 0 & 0 & p_3 \end{pmatrix}$$

Instead of 3 parameters, we disturb 9 parameters every iteration.

Use quantum relative entropy instead of fidelity as objective function:

$$f\left(\sigma_{\pm,k}\right) = \sum_i p_{\pm,k,i} \log\left(v_{\pm,k,i}\right)$$



Blue: low statistical noise (N~800k counts)

Green: high statistical noise (N~800 counts)

Try self-guided tomography in other physical systems! (We can share code.)

https://arxiv.org/abs/2010.00632

SGT with frequency qudits

IOP Publishing Quantum Sci. Technol. 10 (2025) 025024 https://doi.org/1(

Quantum Science and Technology

PAPER

Self-guided tomography of time-frequency qudits

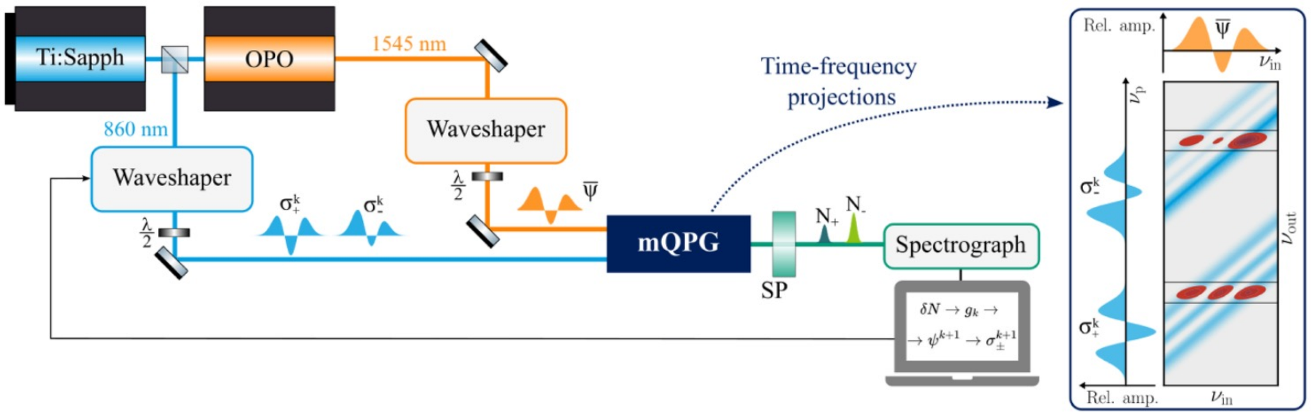
Laura Serino[1,*], Markus Rambach[2,3], Benjamin Brecht[1], Jacquiline Romero[2,3] and Christine Silberhorn[1]

OPEN ACCESS

RECEIVED
28 November 2024

Above 99% fidelity for d=3 and d=5.

	$d=3$		$d=5$	
N	SGT	MLST	SGT	MLST
10^5	$0.65^{+0.51}_{-0.29}\%$	$0.88^{+0.58}_{-0.47}\%$	$0.94^{+0.54}_{-0.33}\%$	$3.24^{+1.02}_{-0.52}\%$
10^4	$0.60^{+0.38}_{-0.24}\%$	$0.78^{+0.34}_{-0.49}\%$	$1.01^{+0.71}_{-0.36}\%$	$2.67^{+0.79}_{-0.72}\%$
10^3	$0.76^{+0.55}_{-0.35}\%$	$0.48^{+0.88}_{-0.22}\%$	$1.23^{+0.62}_{-0.55}\%$	$3.86^{+0.90}_{-1.02}\%$
10^2	$7.0^{+3.0}_{-3.4}\%$	$13.1^{+8.0}_{-6.0}\%$	$7.9^{+3.7}_{-3.0}\%$	$39.6^{+4.7}_{-5.3}\%$



Plan…

Review photonic qudits, focus on transverse mode (shape)

Convince you that ignorance of the whole does not imply ignorance of the parts.

Convince you that self-guided tomography is a robust and efficient way to do quantum state tomography.

Look at a method to implement high-d gates in shape.

Review some photonic entanglement experiments.

	Generation	Processing	Measurement and detection
Free-space	Spatial light modulator (SLM) spontaneous parametric down-conversion (SPDC)	??? Multi-plane light conversion (MPLC)	Spatial light modulator and single photon detection

Joel Carpenter

Daniel Dahl

Qudit gates

Any operator in a *d*-dimensional Hilbert space can
be expressed as,

$$X^j Z^k \qquad j, k \in \mathbb{Z}_d$$

$$X|s\rangle = |s + 1(\text{mod } d)\rangle$$

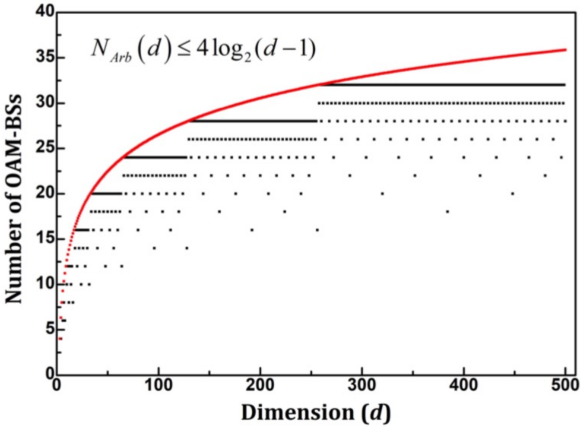
$$Z|s\rangle = \exp(2\pi i s/d)|s\rangle$$

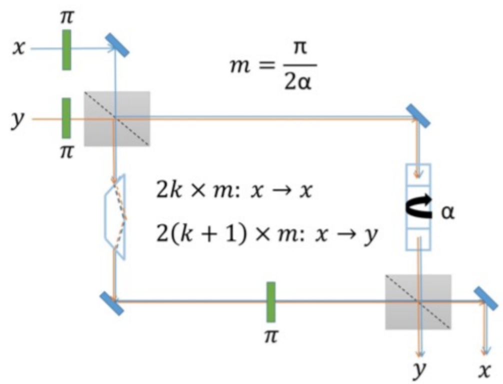
For *d*=3:

$$\hat{X}^1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad \hat{Z}_1 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega_3 & \omega_3^2 \\ 1 & \omega_3^2 & \omega_3 \end{bmatrix}$$

$$\omega_3 = e^{2\pi i/3}$$

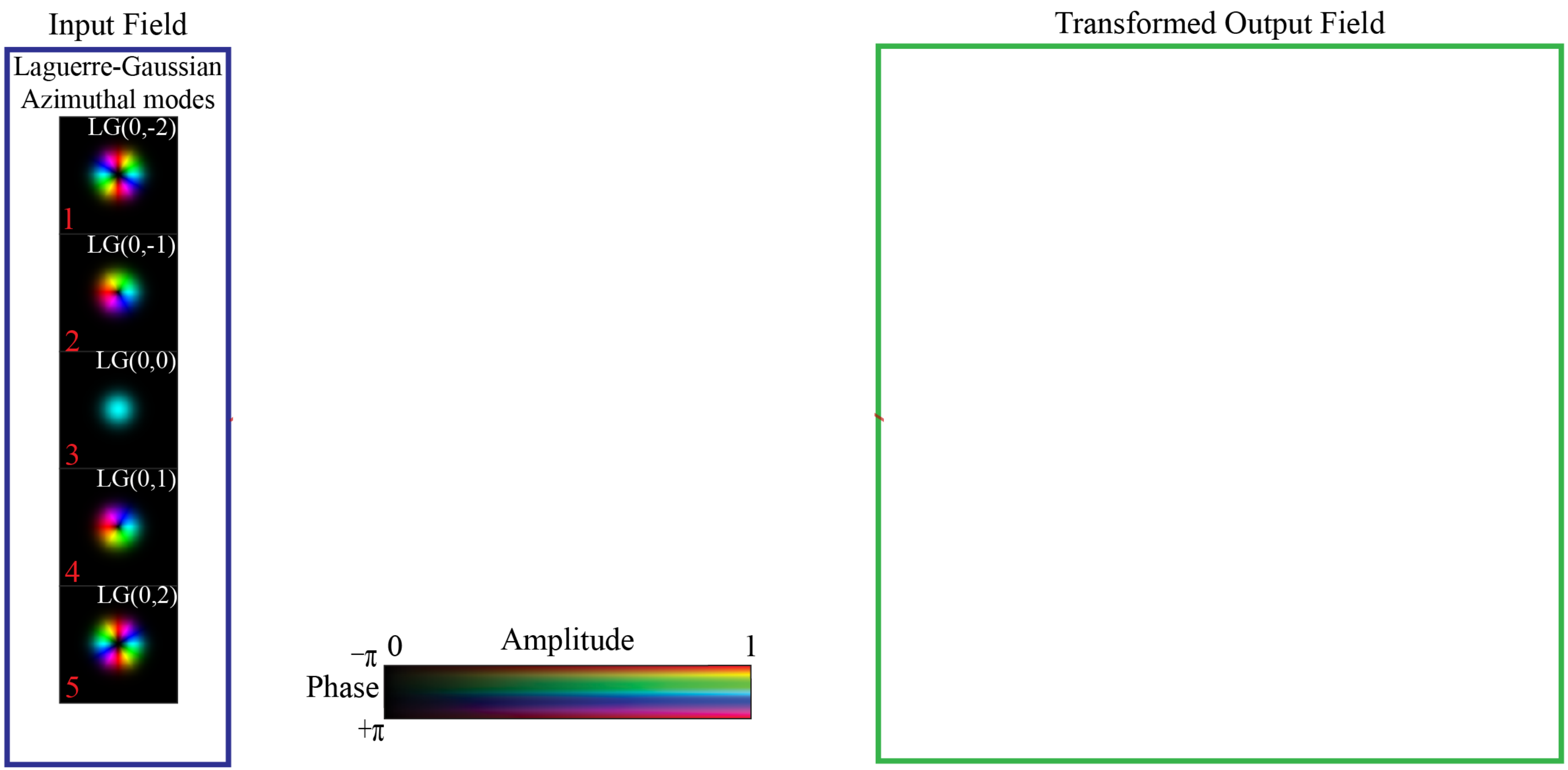
For an X-gate…





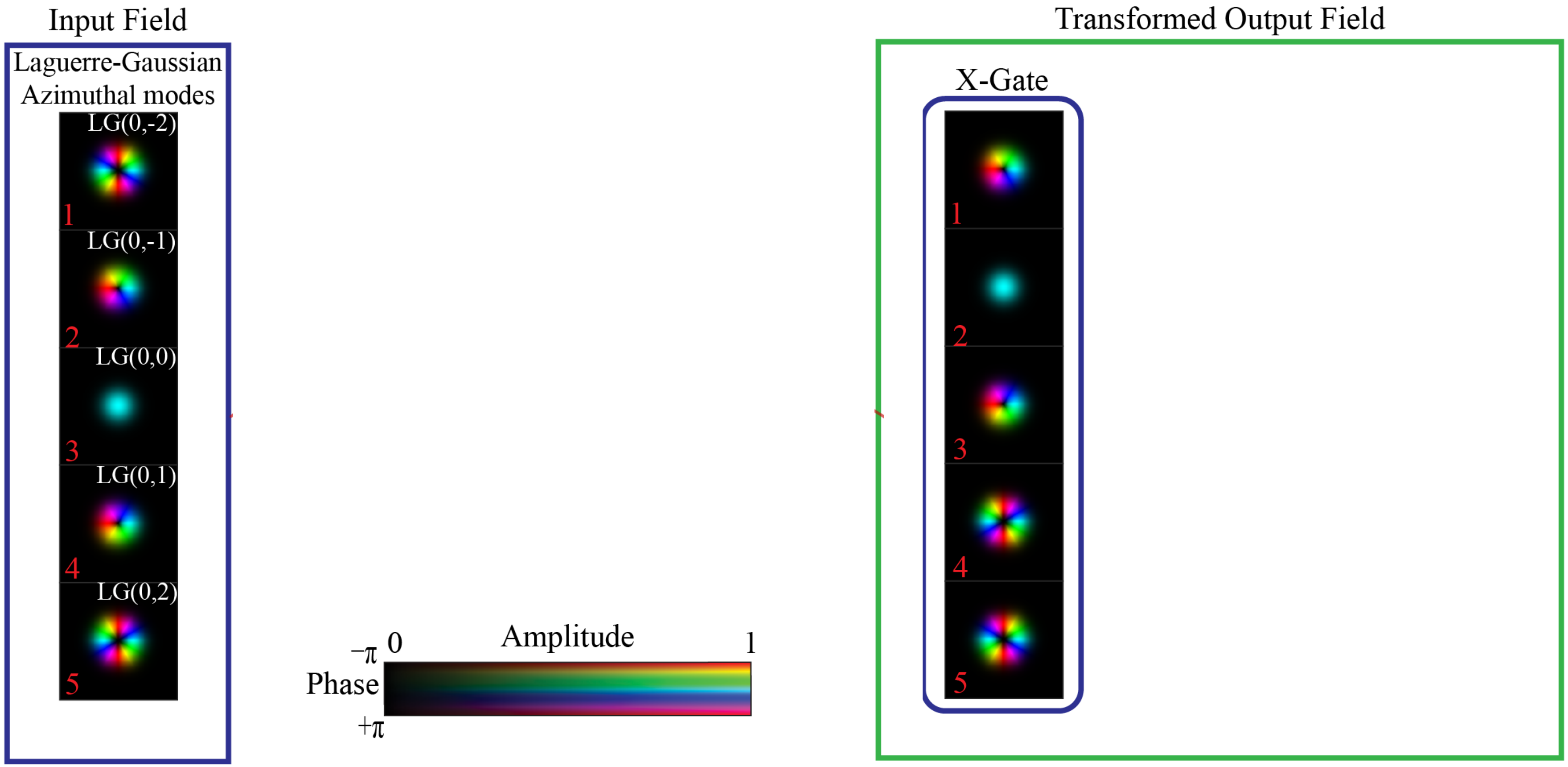
Gao et al., Phys. Rev. A 99, 023825 (2019)

Qudit gates



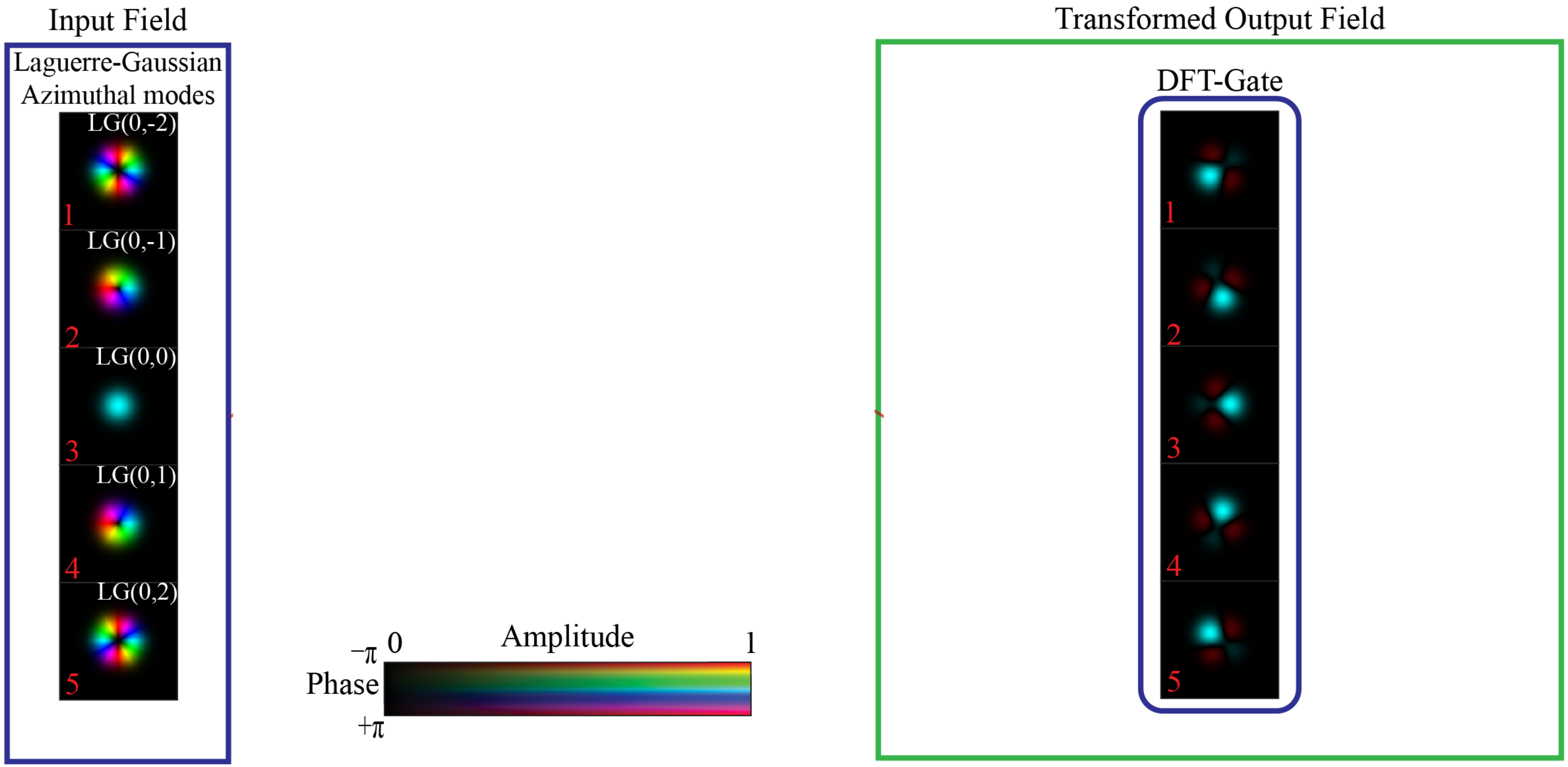
Dahl et al., FiO 2024

Qudit gates



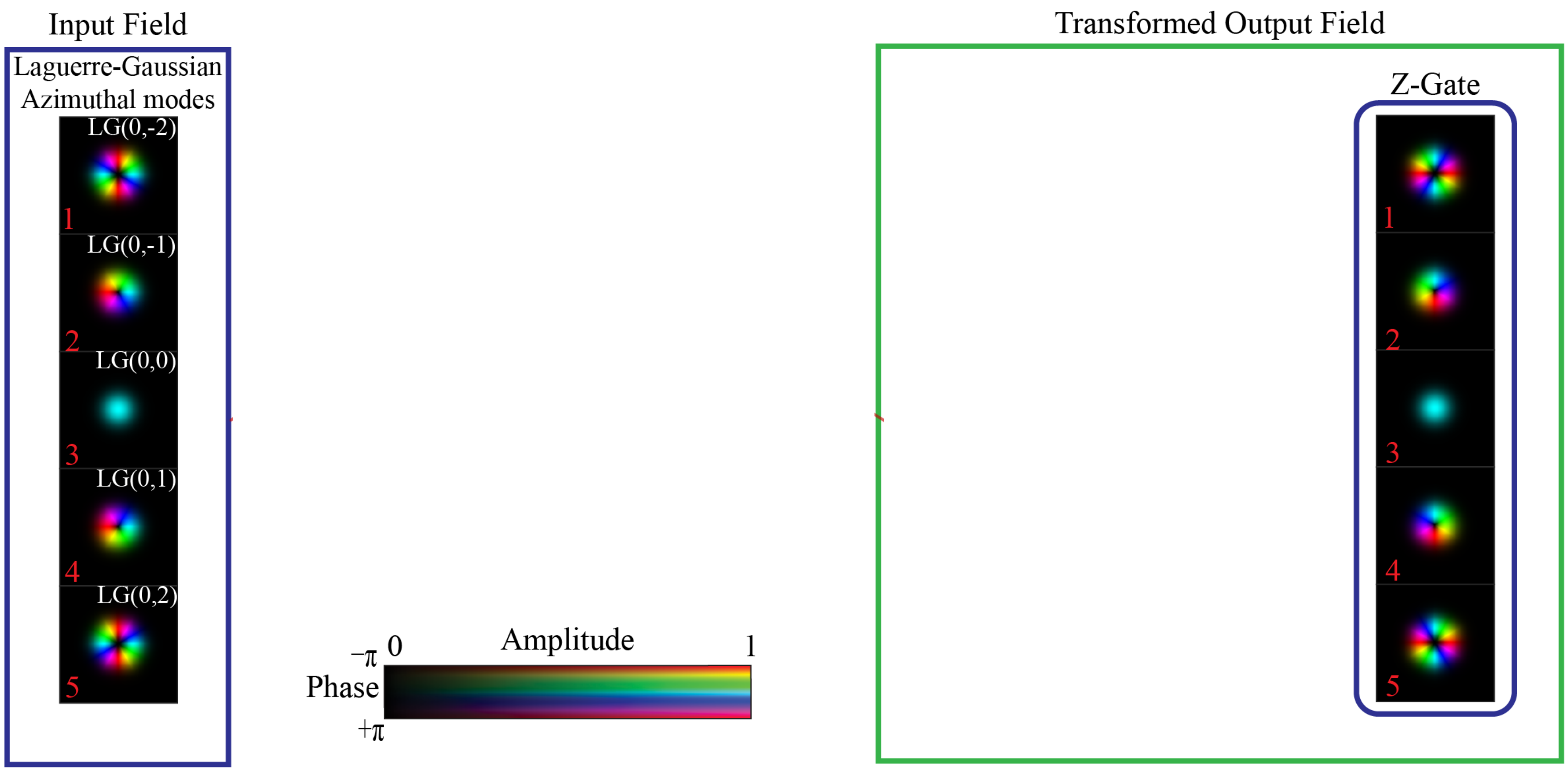
Dahl et al., FiO 2024

Qudit gates



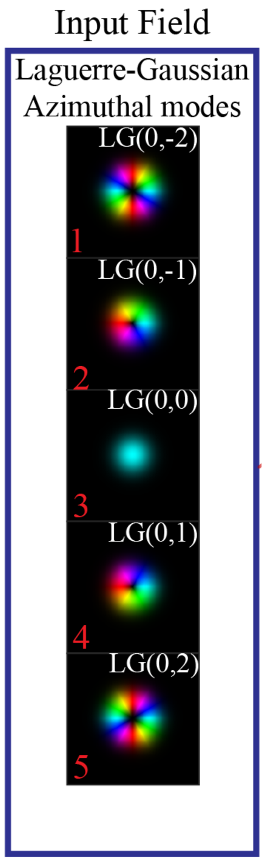
Dahl et al., FiO 2024

Qudit gates



Dahl et al., FiO 2024

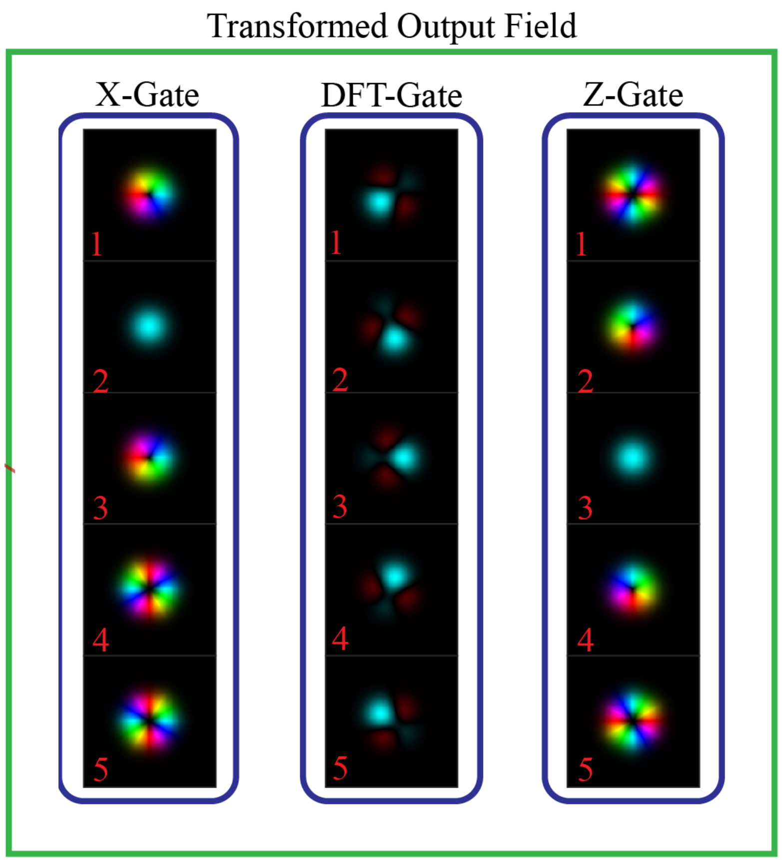
Qudit gates



Design by specifiying input and output

??

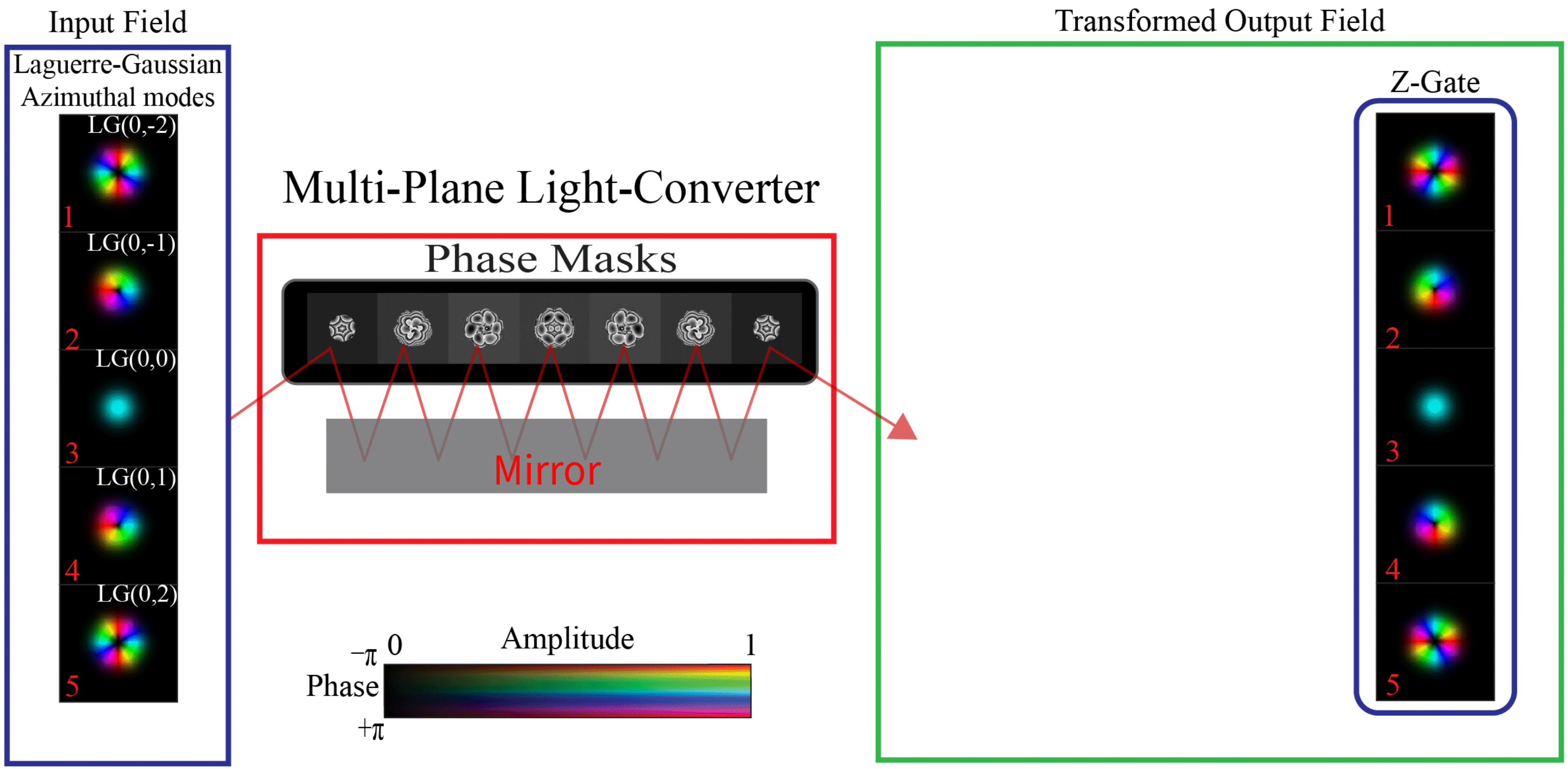
?



Sakamaki et al., J Lightwave Technology 5, 3511 (2007)

Dahl et al., FiO 2024

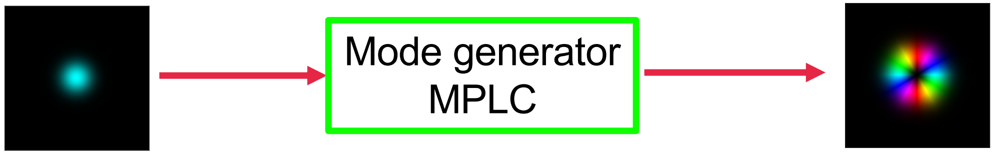
Qudit gates via MPLC

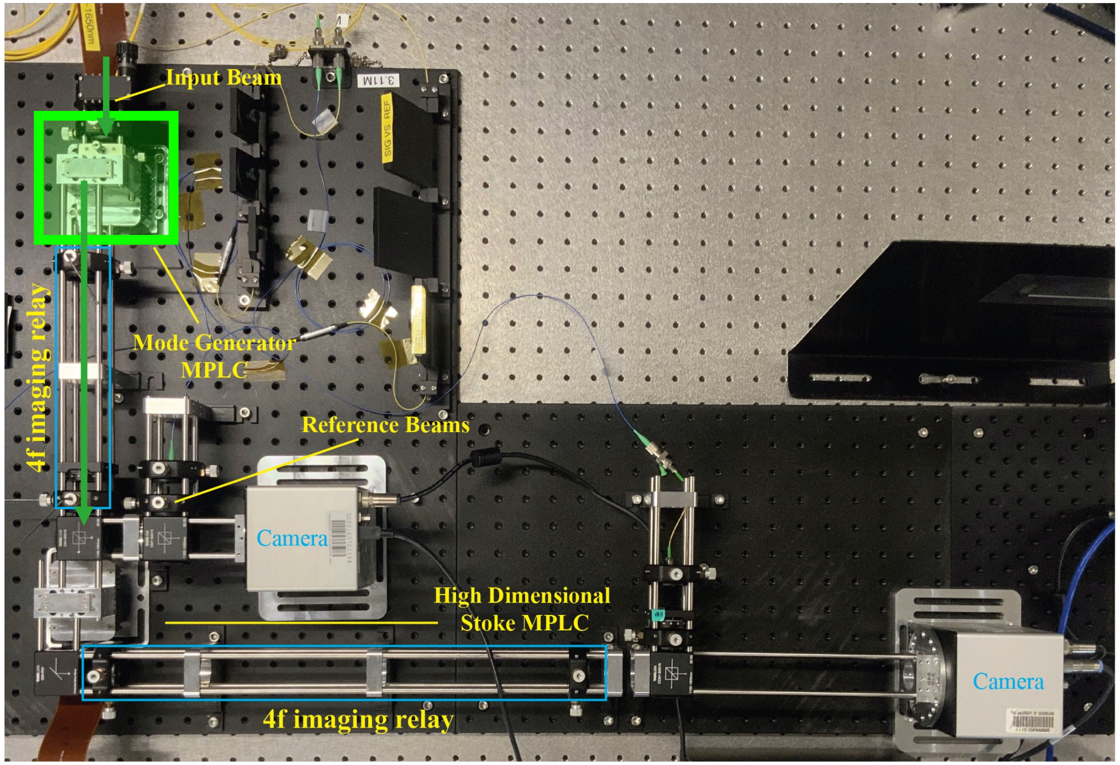


Sakamaki et al., J Lightwave Technology 5, 3511 (2007)

Dahl et al., FiO 2024

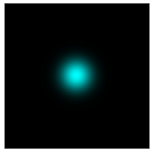
Experiment setup



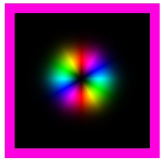


Dahl et al., FiO 2024

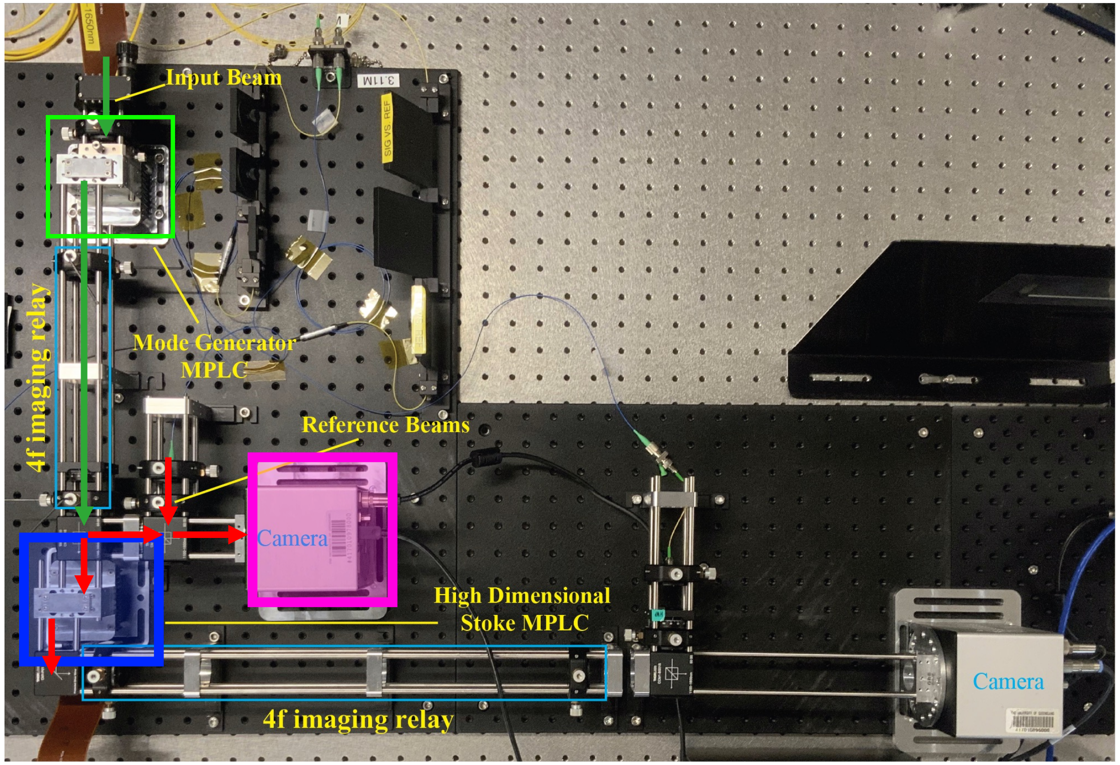
Experiment setup



Mode generator MPLC

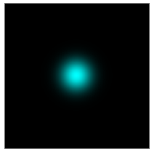


Qudit Gate MPLC

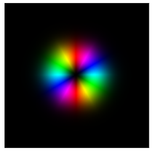


Dahl et al., FiO 2024

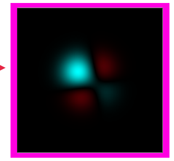
Experiment setup

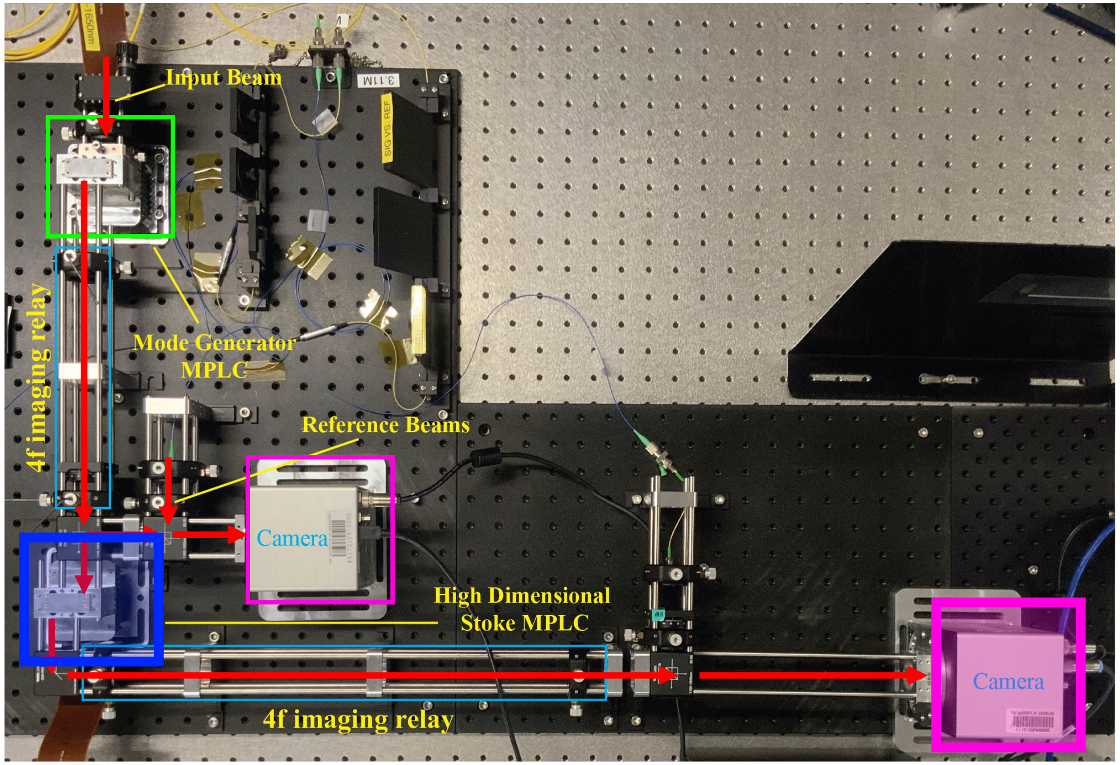


Mode generator MPLC



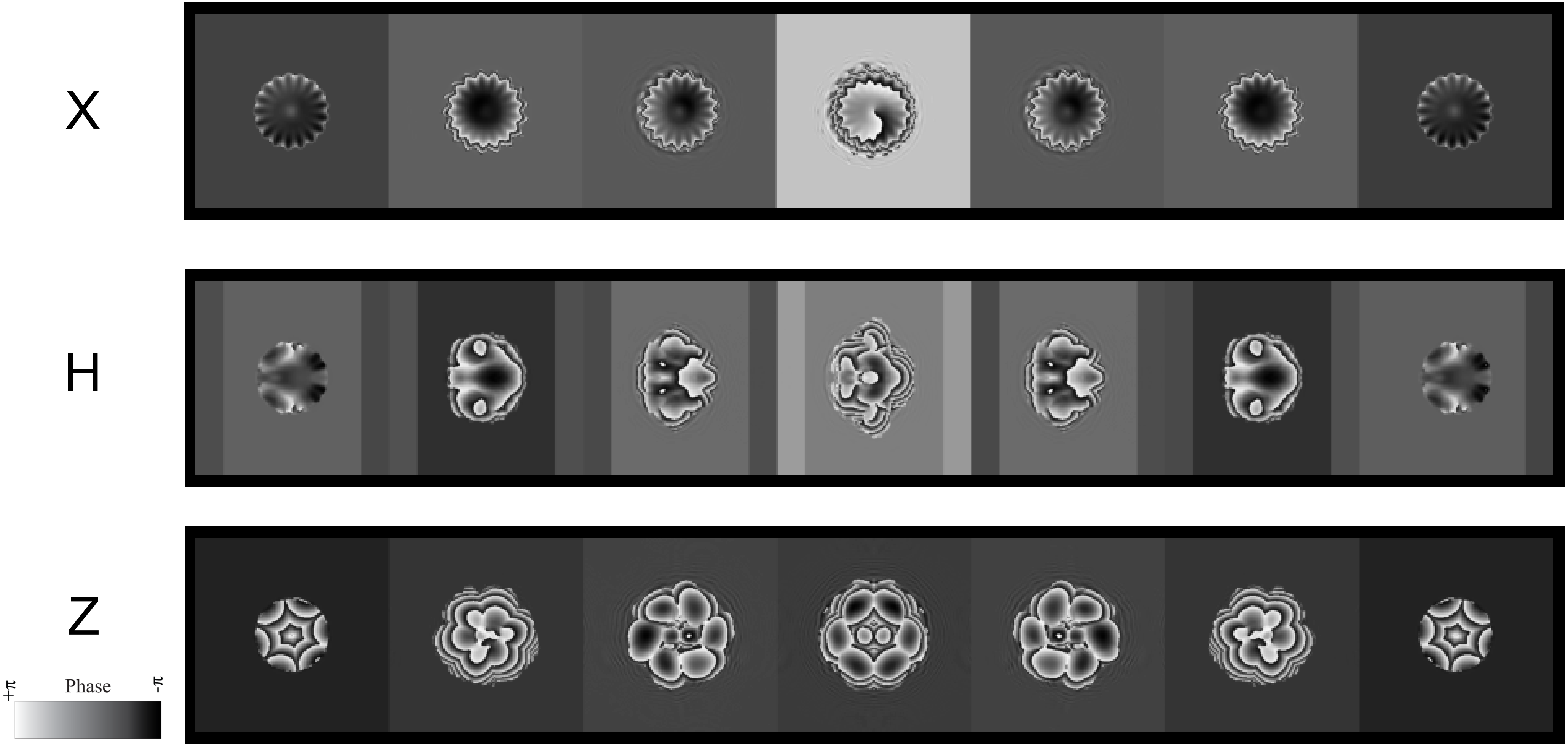
Qudit Gate MPLC



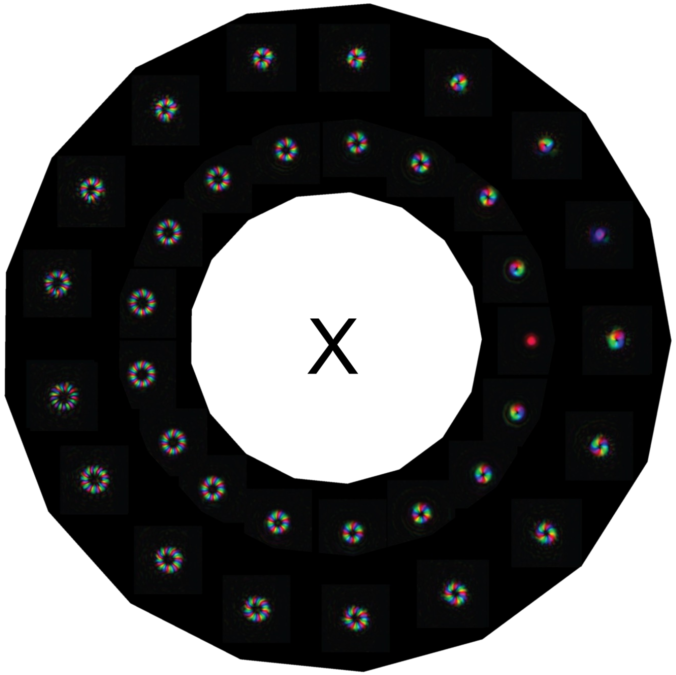


Dahl et al., FiO 2024

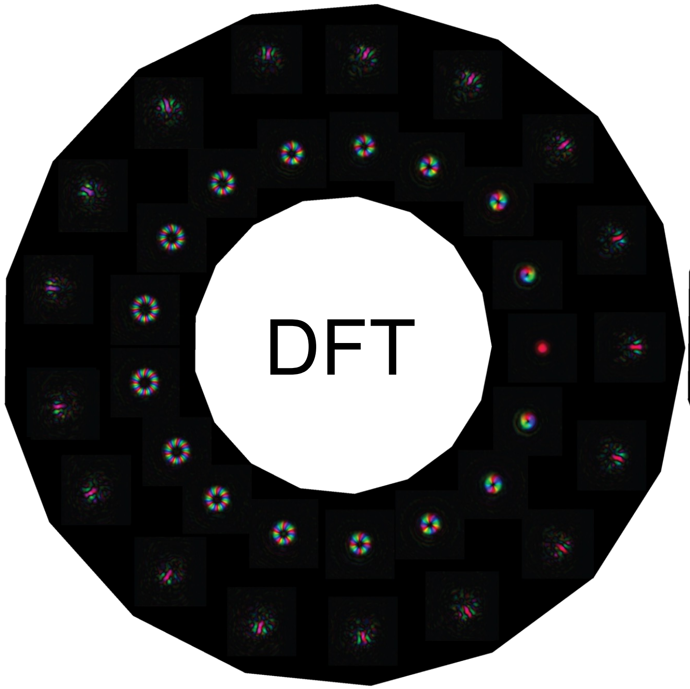
Qudit gates via MPLC (d=17)



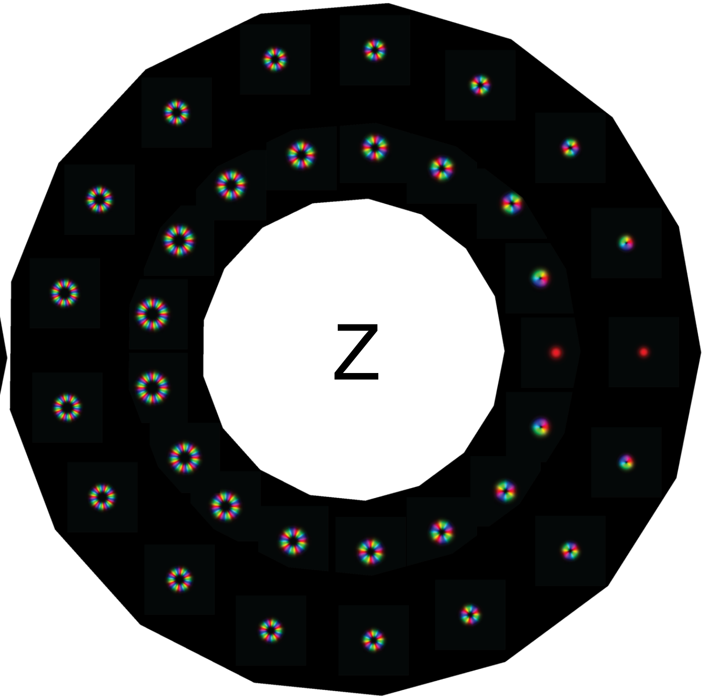
Results: Fields and transfer matrix



IL= -9.4dB (88.5% loss)
MDL= 4.67 dB
Visibility = 0.99
SNR=16.58 dB



IL = -9.4dB (88.5% loss)
MDL= 5.63 dB
Visibility = 0.93
SNR=12.90 dB



IL = -9.4dB (88.5% loss)
MDL= 3.61 dB
Visibility = 0.96
SNR=13.40 dB

Dahl et al., FiO 2024

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Entangled photons from atomic cascade

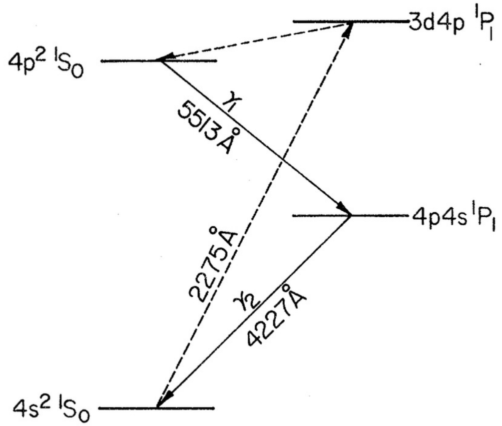
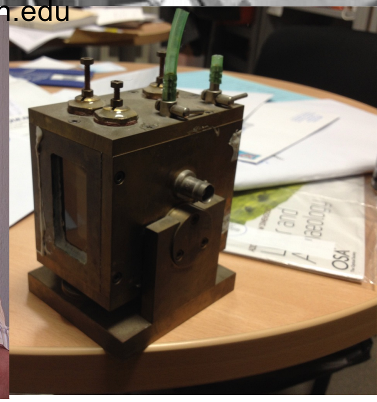
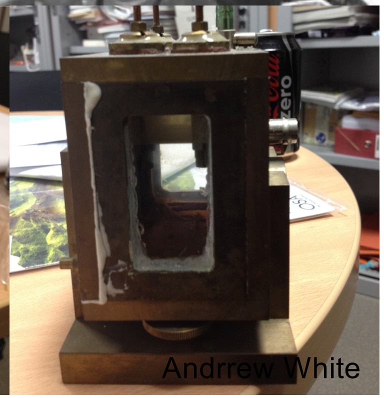


FIG. 2. Level scheme of calcium. Dashed lines show the route for excitation to the initial state $4p^2\ ^1S_0$.





Freedman and Clauser, Phys. Rev. Lett 28 (1972)

Aspect et al., Phys. Rev. Lett 49 (1982)

Entangled photons from nonlinear optics

VOLUME 61, NUMBER 26 PHYSICAL REVIEW LETTERS 26 DECEMBER 1988

New Type of Einstein-Podolsky-Rosen-Bohm Experiment Using Pairs of Light Quanta Produced by Optical Parametric Down Conversion

Y. H. Shih and C. O. Alley
Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742
(Received 23 May 1988)

A pair of correlated light quanta of 532-nm wavelength with the same linear polarization but divergent directions of propagation was produced by nonlinear optical parametric down conversion. Each light quantum was converted to a definite polarization eigenstate and was reflected by a turning mirror to superpose with the other at a beam splitter. For coincident detection at separated detectors, polarization correlations of the Einstein-Podolsky-Rosen-Bohm type were observed. We also observed a violation of Bell's inequality by 3 standard deviations.

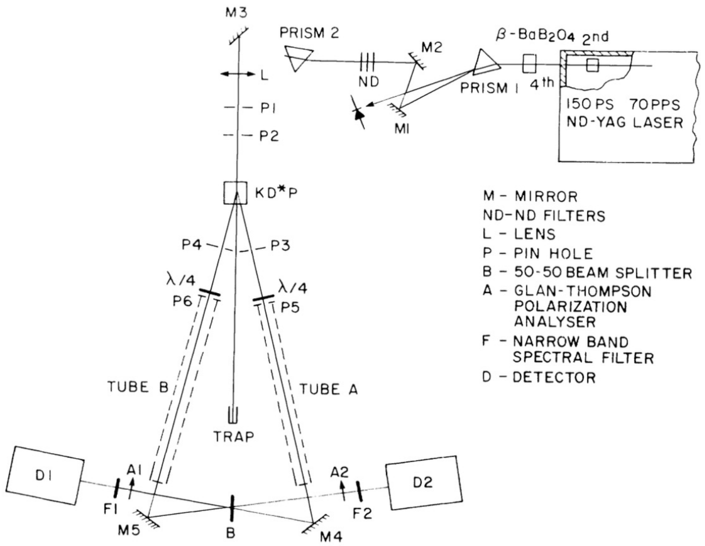
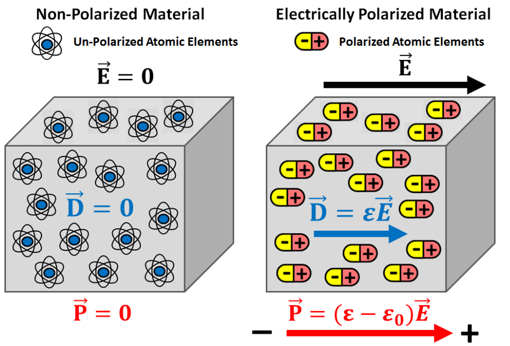


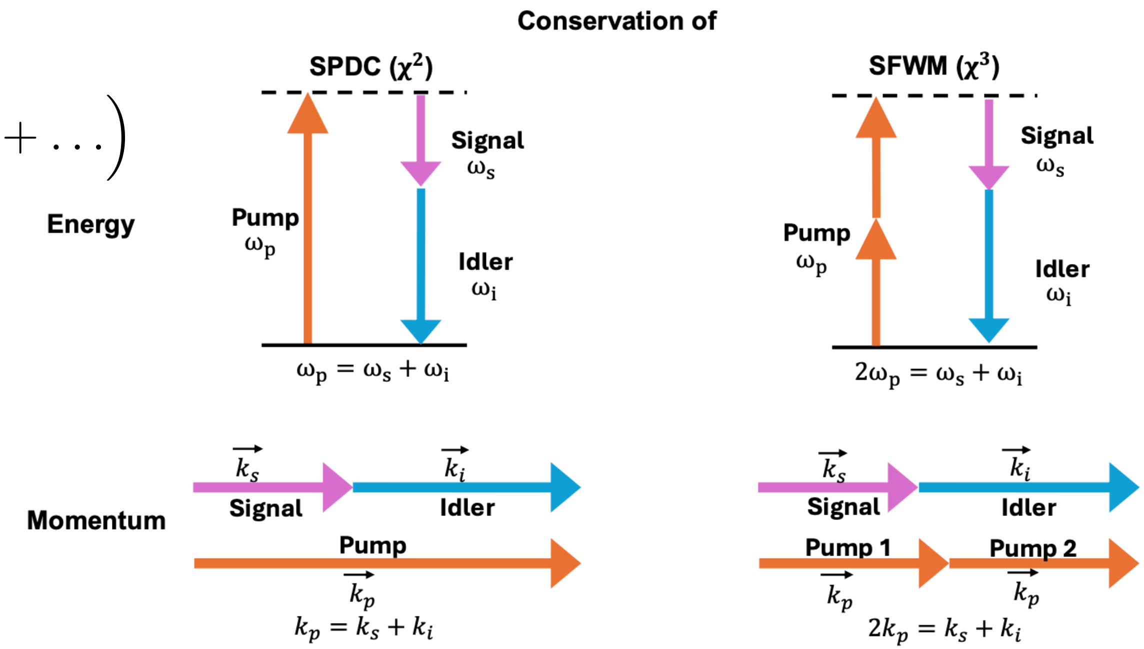
FIG. 1. Schematic diagram of the experiment.

Entangled photons from SPDC and SFWM

Conservation of transverse momentum and energy lead to entanled pairs of photons. (Multi-photon events also possible, but avoided.)

$$\mathbf{P} = \mathbf{P}^{(1)} + \mathbf{P}^{(2)} + \mathbf{P}^{(3)} \dots$$
$$= \epsilon_0 \left(\chi^{(1)}\mathbf{E} + \chi^{(2)}\mathbf{E^2} + \chi^{(3)}\mathbf{E^3} + \dots \right)$$
$$= \mathbf{P}^{(1)} + \mathbf{P}_{\text{nonlinear}}$$





Bell (CGLMP) inequality violations in higher dimensions

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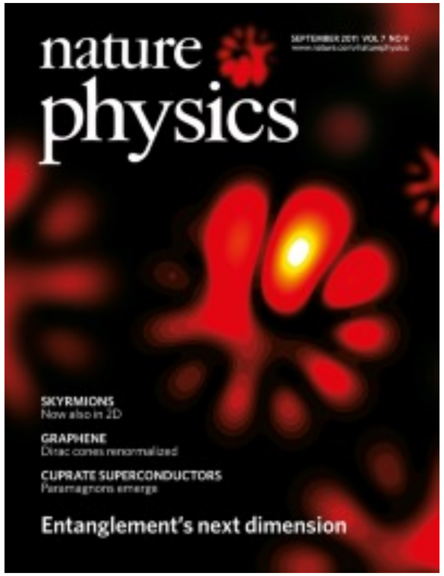
Letter | Published: 08 May 2011

Experimental high-dimensional two-photon entanglement and violations of generalized Bell inequalities

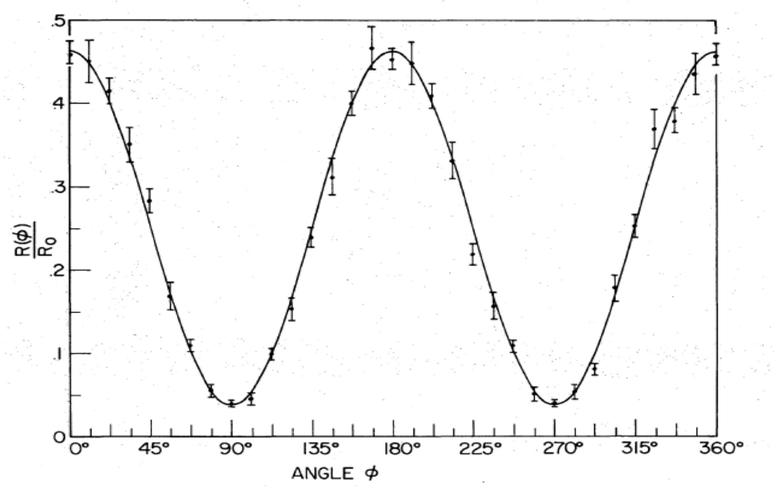
Adetunmise C. Dada, Jonathan Leach, Gerald S. Buller, Miles J. Padgett & Erika Andersson

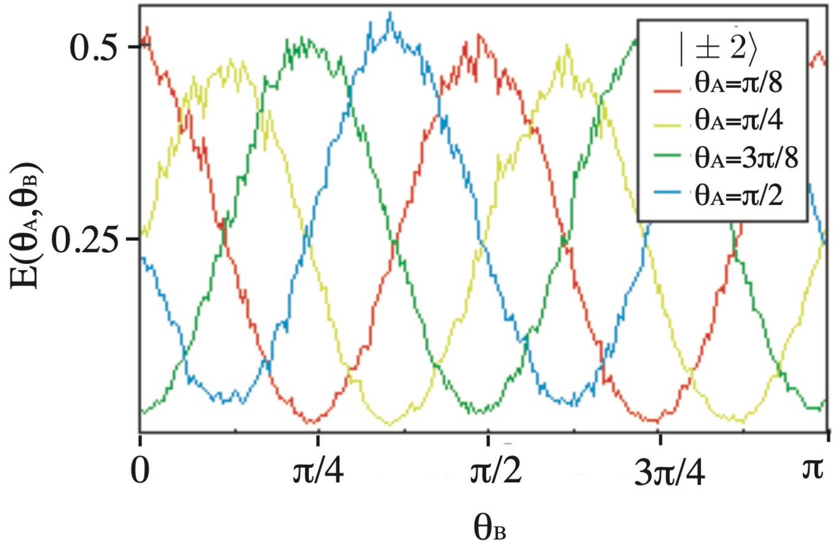
Nature Physics **7**, 677–680 (2011) | Cite this article



Bell (CGLMP) inequality violations in higher dimensions

Two-dimensional subspaces violate the Bell inequality.





Freedman and Clauser, Phys. Rev. Lett 28 (1972)

Jack et al. Optics Exp. 17 (2009)

Bell (CGLMP) inequality violations in higher dimensions

State:

$$|\Phi\rangle = \frac{1}{\sqrt{d}} \sum_{\ell=-[d/2]}^{[d/2]} h(\ell)|\ell\rangle_A \otimes |-\ell\rangle_B$$

Measurements:

$$|v\rangle_a^A = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} \exp\left[i\frac{2\pi}{d}j\left(v+\alpha_a\right)\right]|j\rangle$$

$$|w\rangle_b^B = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} \exp\left[i\frac{2\pi}{d}j\left(-w+\beta_b\right)\right]|j\rangle$$

$\alpha_0 = 0, \alpha_1 = 1/2, \beta_0 = 1/4$ and $\beta_1 = -1/4.$

Translate to experiment measurement:

$$|v\rangle_a^A \equiv |\theta_A^a\rangle = \frac{1}{\sqrt{d}} \sum_{\ell=-[d/2]}^{\ell=+[d/2]} \exp\left[i\theta_A^a g(\ell)\right]|\ell\rangle, \text{ and}$$

$$|w\rangle_b^B \equiv \left|\theta_B^b\right\rangle = \frac{1}{\sqrt{d}} \sum_{\ell=-\left[\frac{d}{2}\right]}^{\ell=+\left[\frac{d}{2}\right]} \exp\left[i\theta_B^b g(\ell)\right]|\ell\rangle$$

where

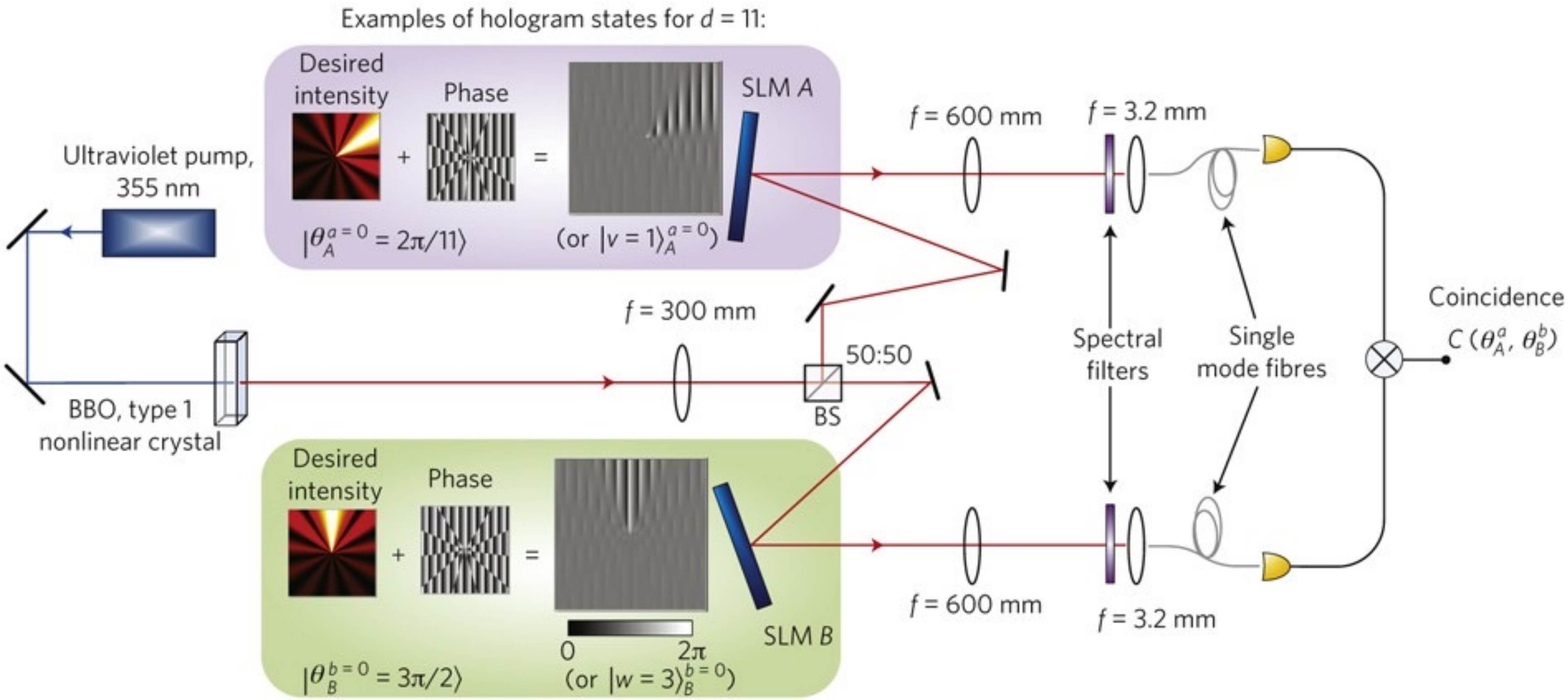
$$\theta_A^a = (v + a/2)2\pi/d$$
$$\theta_B^b = \left[-w + 1/4(-1)^b\right]2\pi/d$$

The function $g(\ell)$ is defined as

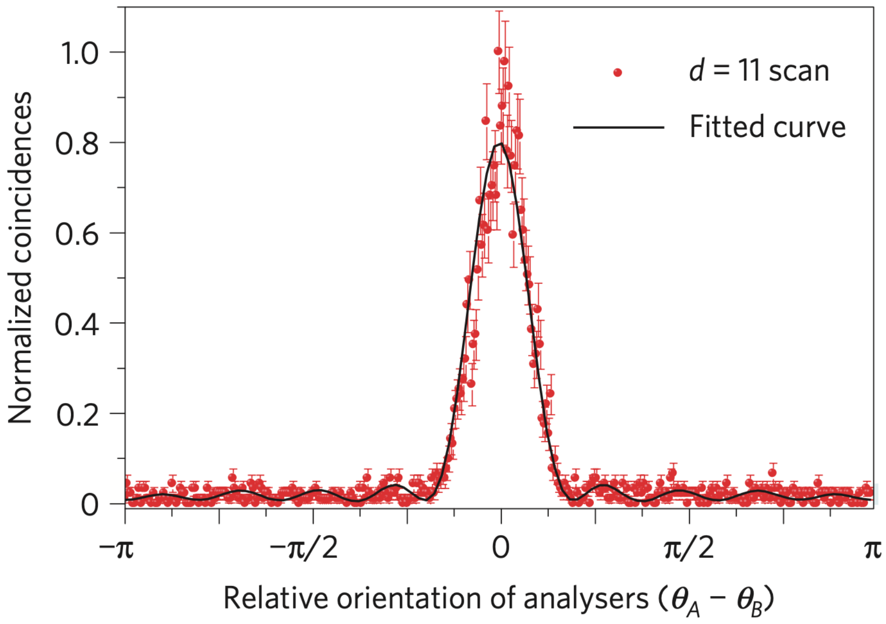
$$g(\ell) = \ell + \left[\frac{d}{2}\right] + (d \bmod 2)u(\ell)$$

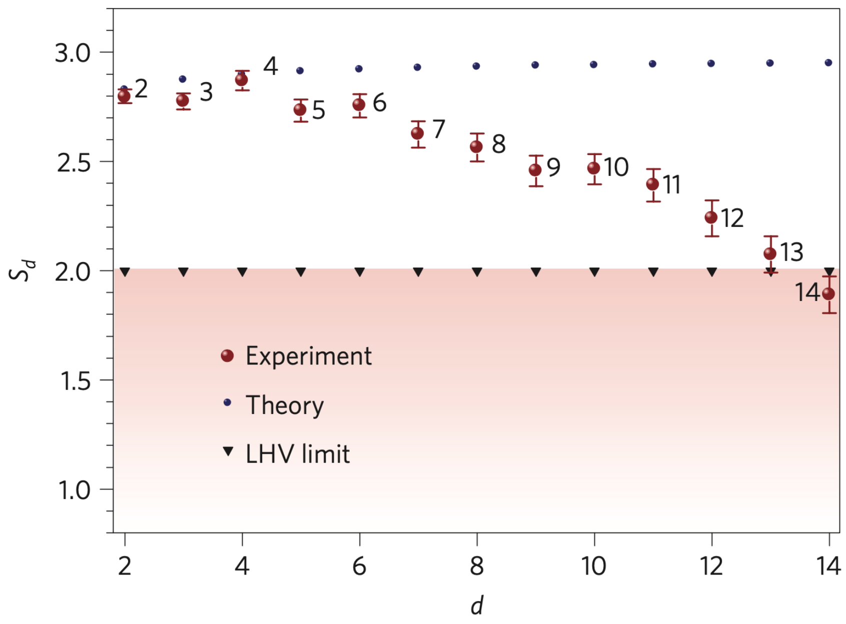
where $[x]$ is the integer part of x, and $\mathrm{u}(\ell)$ is the discrete unit step function.

Bell (CGLMP) inequality violations in higher dimensions



Bell (CGLMP) inequality violations in higher dimensions



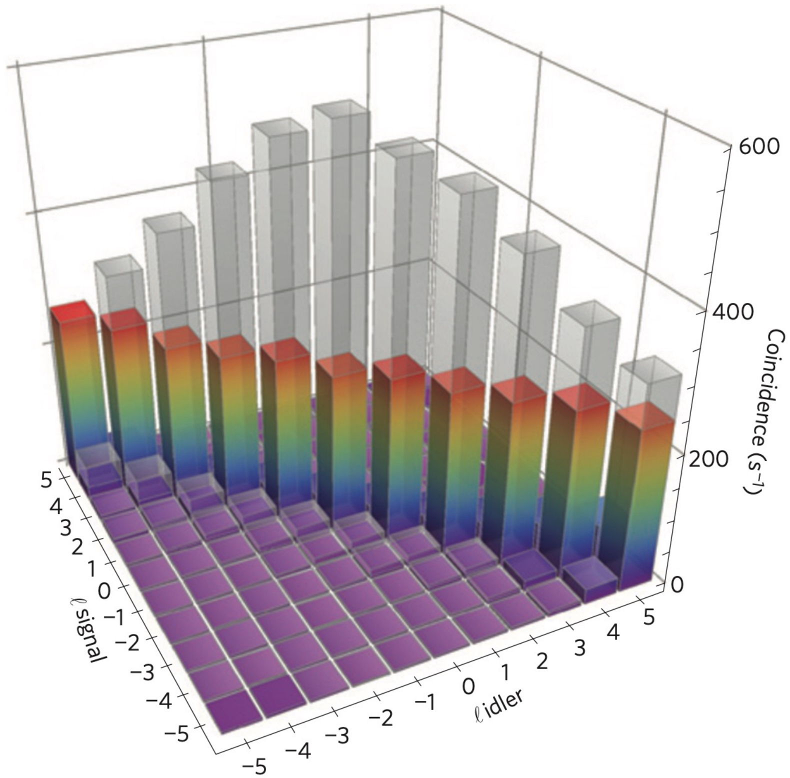


Correlations (coincidences) are
no longer sinusoidal.

Bell (CGLMP) inequality violations in higher dimensions

Generating maximally entangled states gets harder to generate as d increases.

Engineering phase-matching will help.

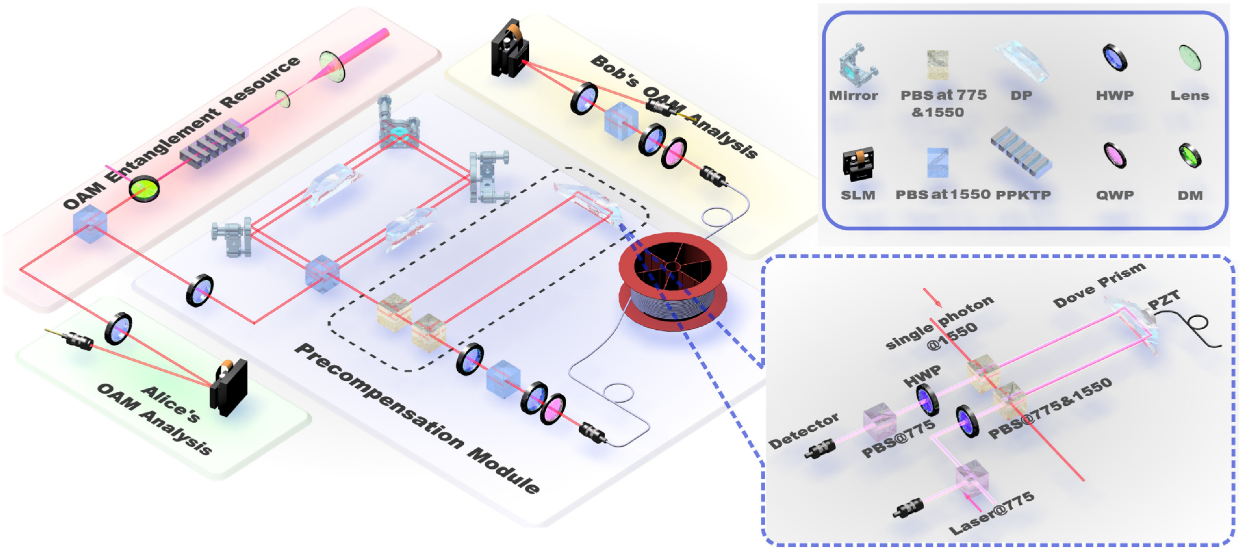


Bell violation over 1 km few-mode optical fibre

Optica Vol. 7, Issue 3, pp. 232-237 (2020) • https://doi.org/10.1364/OPTICA.381403

Distribution of high-dimensional orbital angular momentum entanglement over a 1 km few-mode fiber

Huan Cao, She-Cheng Gao, Chao Zhang, Jian Wang, De-Yong He, Bi-Heng Liu, Zheng-Wei Zhou, Yu-Jie Chen, Zhao-Hui Li, Si-Yuan Yu, Jacquiline Romero, Yun-Feng Huang, Chuan-Feng Li, and Guang-Can Guo

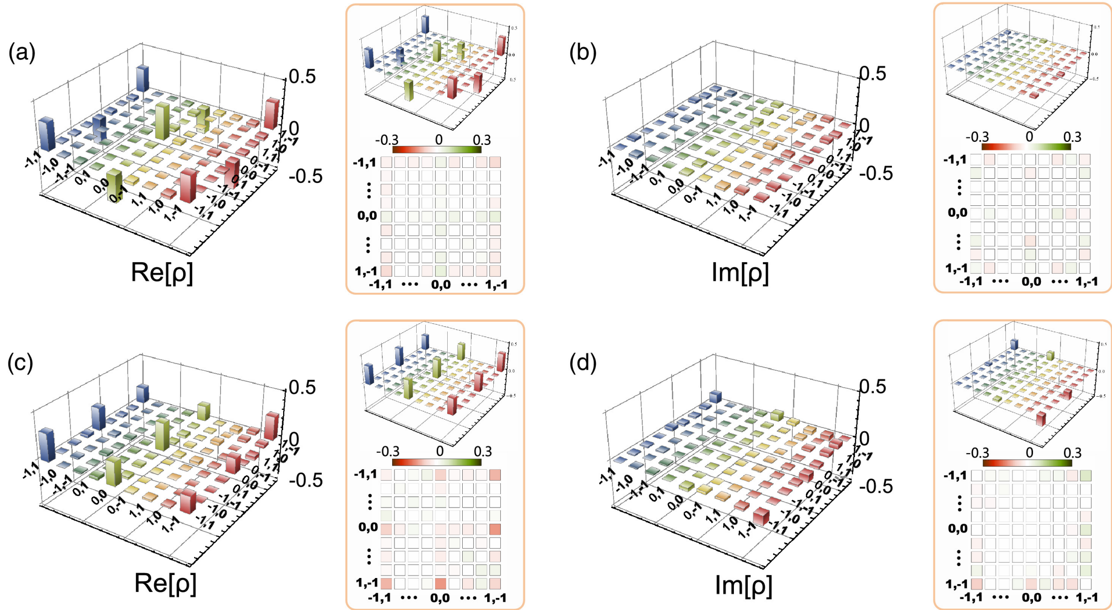


Bell violation over 1 km few-mode optical fibre

$$|\Psi_{\mathrm{MES}}(\theta,\varphi)\rangle = \left(e^{i\theta}|-1\rangle|1\rangle + |0\rangle|0\rangle + e^{i\varphi}|1\rangle|-1\rangle\right)/\sqrt{3}$$

$$I_3 = 2.12 \pm 0.04$$

(it is more challenging to violate after 1 km of fibre, as expected)



Quantum state tomography (71% fidelity)

Multi-photon high-dimensional entanglement

LETTERS
PUBLISHED ONLINE: 29 FEBRUARY 2016 | DOI: 10.1038/NPHOTON.2016.12
nature photonics

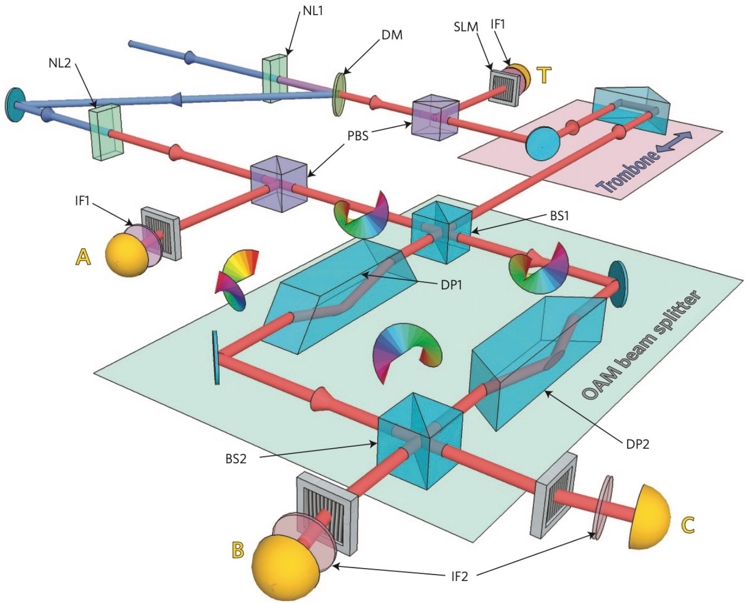
Multi-photon entanglement in high dimensions

Mehul Malik[1,2*], Manuel Erhard[1,2], Marcus Huber[3,4,5], Mario Krenn[1,2], Robert Fickler[1,2†] and Anton Zeilinger[1,2]

$$|\Psi\rangle_{332} = \frac{1}{\sqrt{3}}\left[|0\rangle_A|0\rangle_B|0\rangle_C + |1\rangle_A|1\rangle_B|1\rangle_C + |2\rangle_A|2\rangle_B|1\rangle_C\right]$$

3 x 3 x 2 entanglement

Multi-photon high-dimensional entanglement

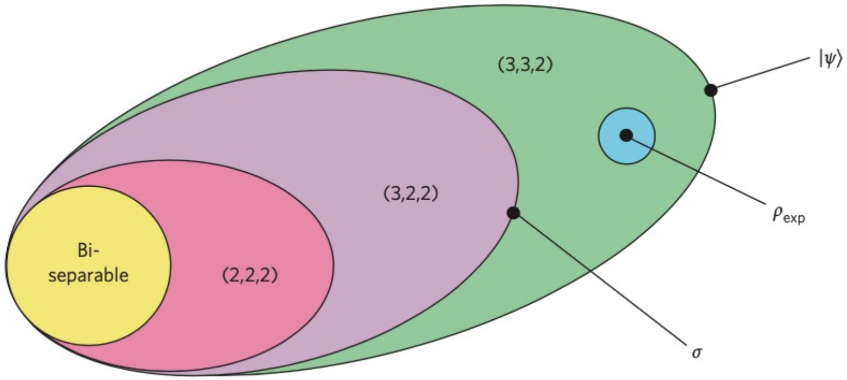


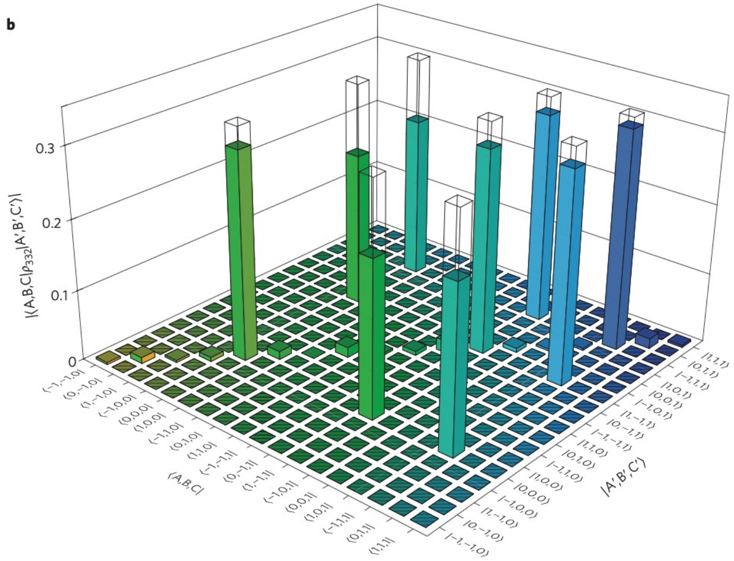
Multiphoton states created from 2 SPDC sources.

4-fold coincidence ~1/min

Multi-photon high-dimensional entanglement

Certify entanglement by entanglement witness.





Estimated density matrix

Take home…

- Photonic qudits based on the transverse shape are great and convenient for exploring qudit quantum information.
- We demonstrate violation of min-entropy splitting inequality. Ignorance of the whole does not imply ignorance of the parts.
- Self-guided tomography is a quick and robust way to do quantum state tomography. Works for mixed states, and has been demonstrated for transver and frequency modes.
- Multi-plane light conversion implements qudit gates, but they are currently very lossy.
- Qudit space is a big space. Using some tricks, photons offers "pristine" states to explore this big space.

Thank you!