GENERAL RELATIVITY HOMEWORK – WEEK 2

Exercise 1. *Consider an electric wire. It consists of positively charged ions at rest, and free electrons flowing in a current. Overall, the wire is electrically neutral, i.e. the total charge density per unit length is* $\lambda = 0$*. The current through the wire is* I*. Now, let us boost to a reference frame at velocity* v *along the direction of the current. What's the charge density* λ *in the new reference frame?*

Exercise 2. *The electromagnetic force law is:*

$$m\alpha^\mu = qF^{\mu\nu}u_\nu \ , \tag{1}$$

where m *is the particle's mass,* q *its electric charge, and* $F^{\mu\nu}$ *is an antisymmetric tensor whose components are the electric and magnetic field strengths:*

$$F^{ti} = -F^{it} = E^i \ ; \quad F^{ij} = \epsilon^{ijk}B_k \ . \tag{2}$$

1. *A particle is moving in a constant electric field* $\mathbf{E} = (E, 0, 0)$*, with initial velocity* $\mathbf{v} = 0$*. Solve the differential equation for the 4-velocity* $u^\mu(\tau)$ *as a function of proper time. Find the spacetime trajectory* $x^\mu(\tau)$*.*

2. *Same, but in a constant magnetic field* $\mathbf{B} = (0, 0, B)$*, with initial velocity* $\mathbf{v} = (v, 0, 0)$*.*