GENERAL RELATIVITY HOMEWORK – WEEK 1

Exercise 1. *Let's play with two applications of our formula* $\epsilon_{ikl}\epsilon_{jkl} = 2\delta_{ij}$.

1. *Consider the Hodge duality* $B_{ij} = \epsilon_{ijk}B_k$. *Find its inversion, i.e. express* B_i *in terms of* B_{ij}.

2. *Consider the formula for the determinant of a matrix* A_{ij}*:*

$$\epsilon_{lmn}A_{il}A_{jm}A_{kn} = (\det A)\epsilon_{ijk} \ . \tag{1}$$

Multiply both sides by a cleverly chosen factor, and deduce a formula for the inverse matrix $(A^{-1})_{ij}$.

Exercise 2. *Consider a particle moving at constant velocity* v_1*, i.e.* $x = v_1 t$.

1. *Boost into a frame moving with velocity* $-v_2$ *along the* x *axis:*

$$t \to \frac{t + v_2 x}{\sqrt{1 - v_2^2}} \ ; \quad x \to \frac{x + v_2 t}{\sqrt{1 - v_2^2}} \ . \tag{2}$$

What is the particle's velocity in the new frame? Compare with the formula for $\tanh(\theta_1 + \theta_2)$.

2. *What will the particle's velocity become if we boost along the* y *axis instead?*