Causality of higher-spin interactions — lightcone and de Sitter space

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Related past work: 2506.16707, 2404.18589, 2303.17866, 2105.07572
[with Julian Lang, Mirian Tsulaia & Emil Albrychiewicz]
Main paper I’m studying: 1807.07542 [Ruslan Metsaev]

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Plan of the talk

- Strange interests: de Sitter static patch, higher spins
- Massless cubic vertices & chirality
- Lightcone formalism in Minkowski and AdS
- Extension to de Sitter & bulk light cones
- A causal property
- Application to static-patch scattering
- Outlook

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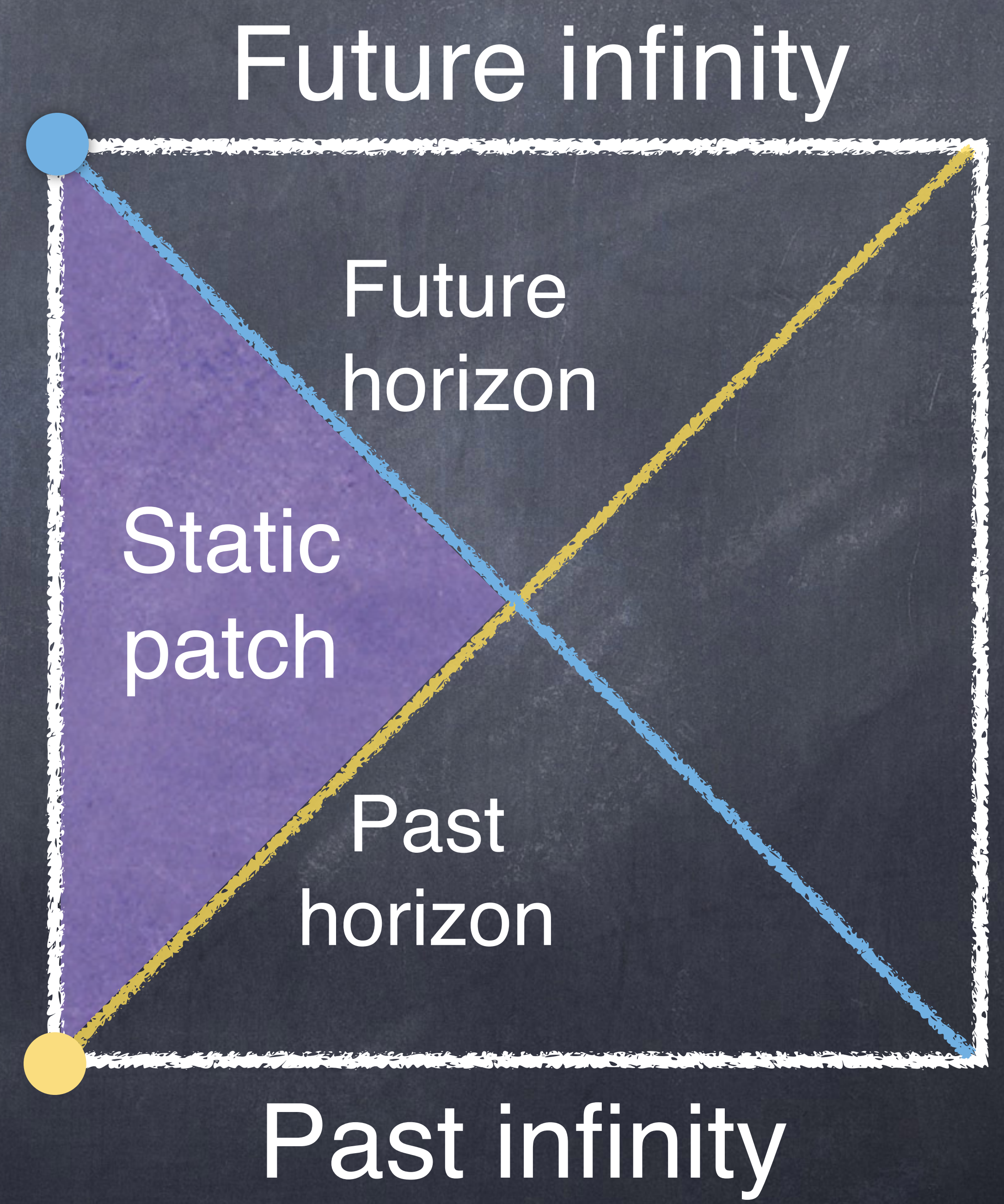
What I want out of life

- Spacetime dimension $D = 3 + 1$
- Quantum Mechanics $\hbar \neq 0$
- Gravity $m = 0\,,\; s = 2$
- Accelerated expansion $\Lambda > 0$

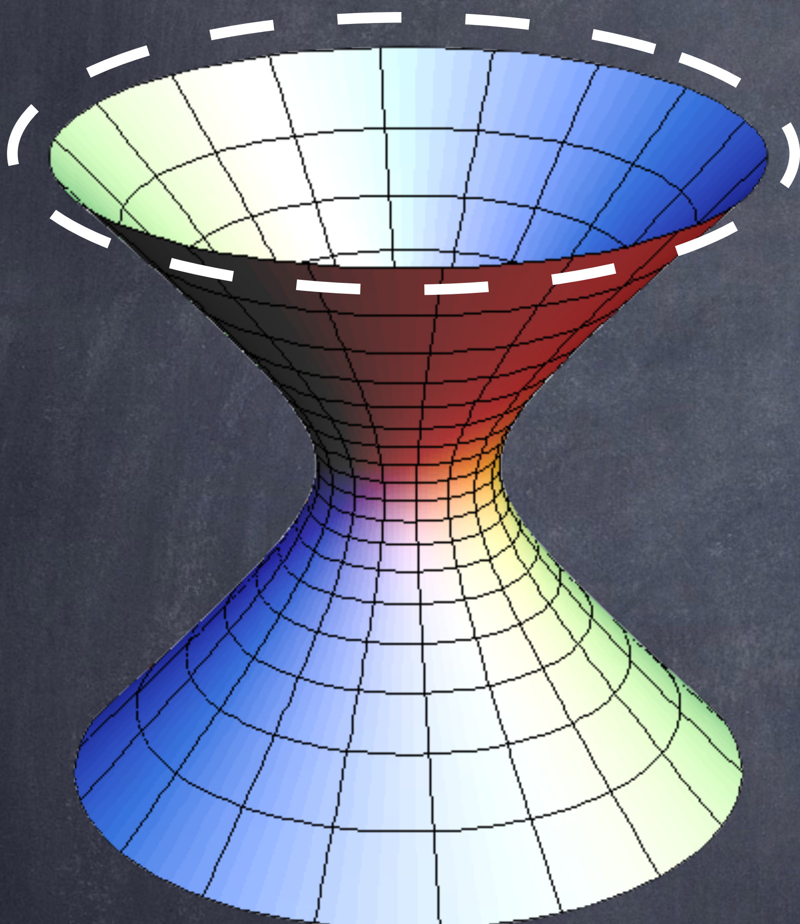
The significance of $\Lambda > 0$

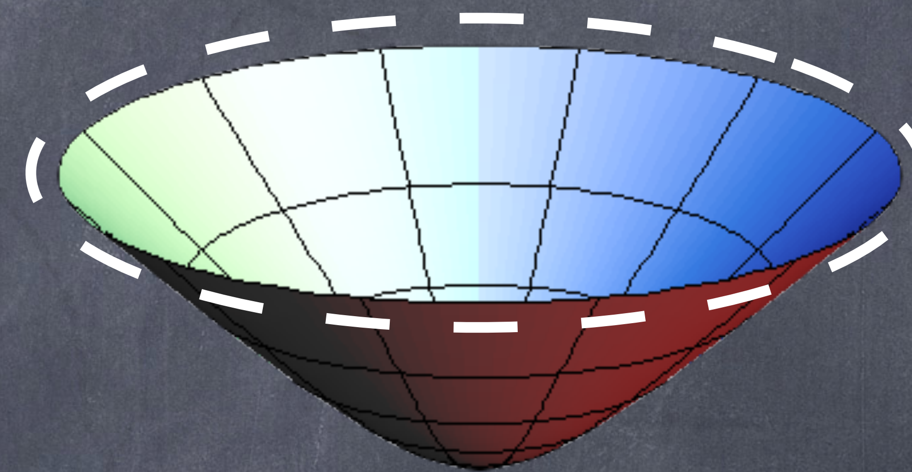
De Sitter space:

- Cosmological horizon around every observer.
- Largest observable region: Static Patch
- Must tackle quantum gravity inside a finite region.



Maldacena-style dS/CFT





dS/CFT $\leftrightarrow$ Euclidean AdS/CFT

Z_{CFT} [sources on boundary] =

= Hartle-Hawking $\Psi_{H.H.}$ in dS [fields at future infinity]

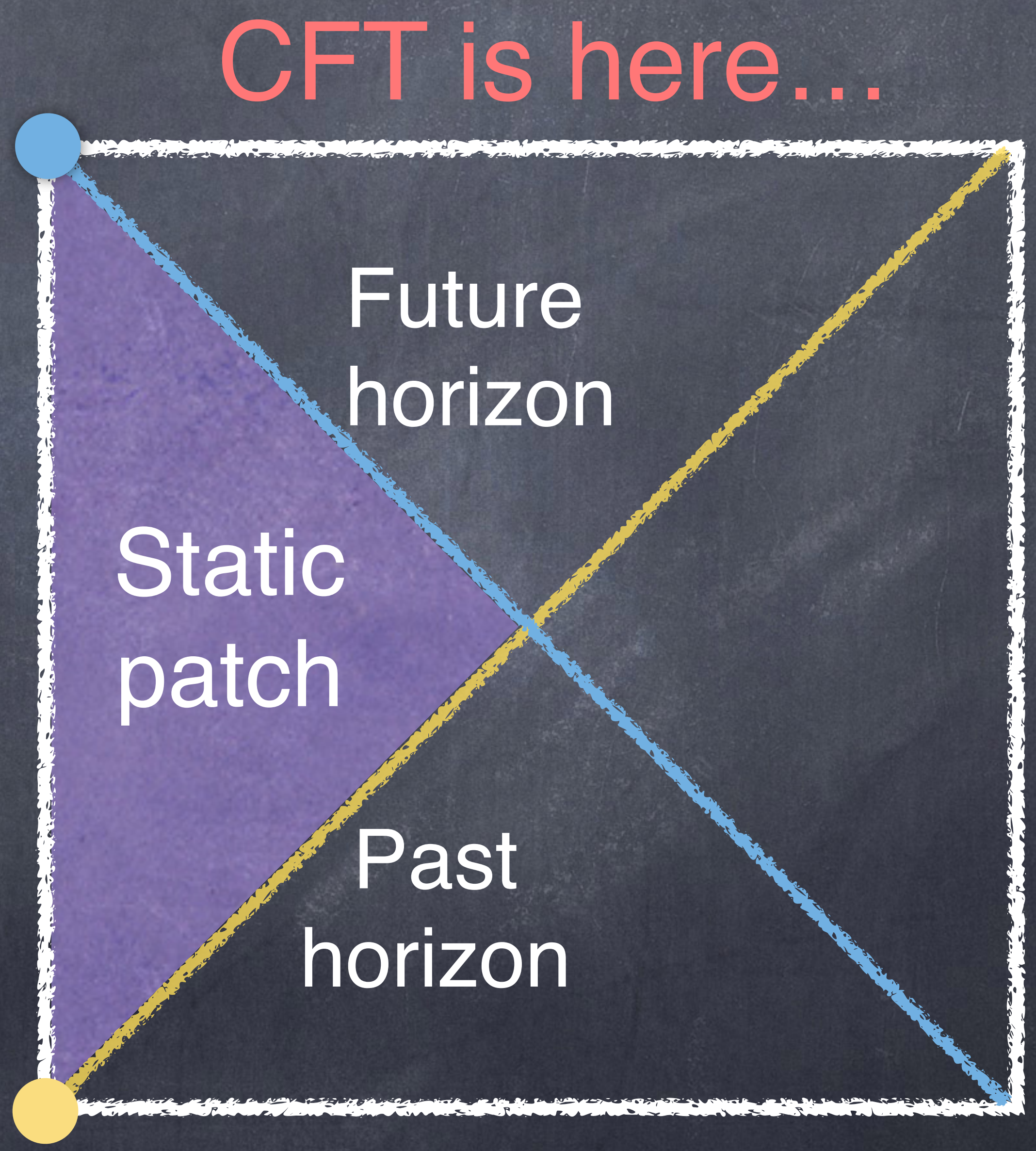
The higher-spin model

4d Bulk: Vasiliev's Higher-Spin Gravity
3d Boundary: N free massless scalars

- AdS/CFT [Klebanov & Polyakov, 2001]
- Survives in dS [Anninos, Hartmann & Strominger, 2011]
- One massless field of every spin:
  $s = 0, 2, 4, 6, \ldots$
- Alternatively, one degree of freedom per helicity:
  $h = 0, \pm 2, \pm 4, \ldots$

Conceptual difficulties

- Find dictionary between future infinity and static patch
- What observabes in static patch?
- CFT lives on spacelike boundary. Time must emerge holographically.



Thus, my strange interests

- Study higher-spin gravity & its holography, especially in de Sitter.
- An observable in the static patch: evolution of field data ("scattering") from past horizon to future horizon
- Try and relate the two.

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Learn to compute in bulk:

* Past work: low-spin interactions (Yang-Mills, self-dual General Relativity)

* This work: higher-spin interactions

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Higher-Spin Gravity

- Spectrum:
  $m = 0\,,\; h = 0, \pm 1, \pm 2, \pm 3, \pm 4, \ldots$

- Cubic vertices: explicitly known in many formalisms (covariant, lightcone, unfolded)

- Quartic+ vertices: only indirect definitions, non-locality

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Classification of massless cubic vertices

- Main property of cubic vertex:
  $h_{\text{tot}} = h_1 + h_2 + h_3$
- $|h_{\text{tot}}| = 1$: Yang-Mills interactions
  (both gluons and matter).
- $|h_{\text{tot}}| = 2$: General Relativity interactions
  (both gravitons and matter).
- $|h_{\text{tot}}| > 2$: "true" higher-spin interactions

Chiral classification of cubic vertices

- $h_{\text{tot}} = 0$: only scalar^3 ($h_1 = h_2 = h_3 = 0$)
  (vanishes in Higher-Spin Gravity)
- $h_{\text{tot}} > 0$: Chiral (includes self-dual YM, GR)
- $h_{\text{tot}} < 0$: Anti-chiral

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Lightcone formalism

- Useful tool for my bulk calculation in de Sitter
- In past work (lower-spin interactions), used lightcone gauge:
  Covariant fields → Lightcone fields
- For higher-spin interactions, easier to use lightcone directly (skip covariant picture)
- Problems to solve:
  1) AdS → dS,
  2) Causality of higher-spin interactions

Unified framework for Minkowski, AdS, dS

- Use Poincare coordinates x^μ, raised/lowered with Minkowski metric $\eta_{\mu\nu}$

- Actual metric is: $\eta_{\mu\nu}$ / $\dfrac{\eta_{\mu\nu}}{z^2}$ / $\dfrac{\eta_{\mu\nu}}{t^2}$

  (flat / AdS / dS)

- Start with 4d conformal symmetry SO(4,2) → reduce to ISO(3,1) / SO(3,2) / SO(4,1)

Lightcone formalism

- Choose special lightlike direction l^μ
- Foliate spacetime into hyperplanes $l \cdot x =$ const
- Single scalar field $\phi_h(x)$ for every helicity h
  — no complicated tensors $\phi_{\mu_1\mu_2\ldots\mu_s}(x)$
- For chiral interactions, convenient to insert asymmetry between positive/negative helicities: $\phi_h \to (\ell \cdot \partial)^{-h}\phi_h$

Lightcone formalism

Action:

$$S = \int d^4x \left(\frac{1}{2} \sum_h \phi_{-h} \Box \phi_h + \sum_{h_{\rm tot}>0} V_{h_1,h_2,h_3} \phi_{h_1} \phi_{h_2} \phi_{h_3} \right)$$

- In absence of covariant (tensor) fields, symmetry not manifest.
- Must define the symmetry generators that change the lightlike reference frame (and check their algebra!)
- What about causality?

Conformal algebra

(Translations) $P^{\mu} = \partial^{\mu}$

(Lorentz) $J^{\mu\nu} = x^{[\mu}\partial^{\nu]} + M^{\mu\nu}$

(Dilatations) $D = x^{\mu}\partial_{\mu} + \Delta$

(Special) $K^{\mu} = \frac{1}{2}x_{\nu}x^{\nu}\partial^{\mu} - x^{\mu}x^{\nu}\partial_{\nu} - x^{\mu}\Delta + x_{\nu}M^{\nu\mu} + R^{\mu}$

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(Special) $K^\mu = \frac{1}{2} x_\nu x^\nu \partial^\mu - x^\mu x^\nu \partial_\nu - x^\mu \Delta + x_\nu M^{\nu\mu} + R^\mu$

- For massless fields in chiral frame:

  $\Delta = 1 - h \; ; \quad M^{\dot{\alpha}\dot{\beta}} = 0 \text{ (right-handed)} \; ; \quad R^\mu = 0$

- Only non-trivial piece: the left-handed part $M^{\alpha\beta}$ of internal Lorentz

Structure of internal Lorentz rotations

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Spinor indices: $\ell^\mu \to q^\alpha \bar{q}^{\dot{\alpha}}$

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Left-handed internal Lorentz (spin + interactions):

$$M^{\alpha\beta} = \frac{1}{l \cdot \partial}\left(h\bar{q}^{\dot{\alpha}} q^{(\alpha}\partial^{\beta)}{}_{\dot{\alpha}} + q^\alpha q^\beta \frac{\delta}{\delta\phi_{-h}} \sum_{h_1,h_2} M_{-h,h_1,h_2}\phi_{-h}\phi_{h_1}\phi_{h_2} \right)$$

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$\Downarrow$

Need two pieces to define the theory:

V_{h_1,h_2,h_3} and M_{h_1,h_2,h_3}

Self-dual Yang-Mills
($h_{\text{tot}} = 1$; full conformal symmetry)

$$V_{h_1,h_2,h_3} = q^\alpha q^\beta \partial^{(1)}{}_{\alpha\dot{\alpha}} \partial_\beta^{(2)\dot{\alpha}} \equiv \mathcal{P}_{(12)}$$

$$M_{h_1,h_2,h_3} = (3h_2 - h_{\text{tot}})(l \cdot \partial^{(1)}) - (3h_1 - h_{\text{tot}})(l \cdot \partial^{(2)}) \equiv \mathcal{M}$$

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General chiral vertex in Minkowski
($h_{\text{tot}} > 0$; dilatations broken)

$$V_{h_1,h_2,h_3} = \mathcal{P}_{(12)}^{h_{\text{tot}}}$$

$$M_{h_1,h_2,h_3} = \mathcal{M}\,\mathcal{P}_{(12)}^{h_{\text{tot}}-1}$$

AdS version
(Metsaev's masterpiece)

- Introduce warp factor z of metric $\eta_{\mu\nu}/z^2$, such that $l^\mu \partial_\mu z = 0$
- Factors of z restore dilatations, but break all generators with components along z direction
- Resulting symmetry is the 3d conformal group SO(3,2) $\leftrightarrow$ AdS group

AdS version
(Metsaev's masterpiece)

- Factors of z modify/complicate integration by parts
- Can rearrange the interacting contributions as:

$$V_{h_1,h_2,h_3} = z^{h_{\rm tot}-1}(\mathcal{P}_{12}+\mathcal{P}_{23}+\mathcal{P}_{31})$$
$$\times\ (\text{Polynom. of order } h_{\rm tot}-1)(\mathcal{P}_{12},\mathcal{P}_{23},\mathcal{P}_{31})$$
$$M_{h_1,h_2,h_3} = z^{h_{\rm tot}} \times (\text{Polynom. of order } h_{\rm tot})(\mathcal{P}_{12},\mathcal{P}_{23},\mathcal{P}_{31})$$

$$\mathcal{P}_{(ij)} = q^\alpha q^\beta \partial^{(i)}{}_{\alpha\dot\alpha}\partial_\beta^{(j)\dot\alpha}$$

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The problem with dS

- Want to replace $\eta_{\mu\nu}/z^2$ by $\eta_{\mu\nu}/t^2$
- In his AdS construction, Metsaev used $l^\mu\partial_\mu z = 0$
- Cannot have $l^\mu\partial_\mu t = 0$!
  (no lightlike directions on dS boundary)
- Must first generalize AdS formalism to $l^\mu\partial_\mu z \neq 0$

Extending to $l^\mu \partial_\mu z \neq 0$

- Recall $l^\mu = q^\alpha \bar{q}^{\dot{\alpha}}$
- $l^\mu \partial_\mu z = 0 \;\longleftrightarrow\; \bar{q}^{\dot{\alpha}} = q^\alpha \partial_\alpha{}^{\dot{\alpha}} z$: Reality condition
- Replacing $\bar{q}^{\dot{\alpha}}$ by $q^\alpha \partial_\alpha{}^{\dot{\alpha}} z$ defines an analytic continuation away from this reality condition, i.e. away from $l^\mu \partial_\mu z = 0$.

Transition to de Sitter now ~trivial

$$\mathcal{P}_{(ij)} = q^{\alpha} q^{\beta} \partial^{(i)}{}_{\alpha\dot{\alpha}} \partial_{\beta}^{(j)\dot{\alpha}}$$

$$V_{h_1,h_2,h_3} = z^{h_{\rm tot}-1}(P_{12} + P_{23} + P_{31}) \times (\text{Polynom. of order } h_{\rm tot} - 1)(P_{12}, P_{23}, P_{31})$$

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$$\Downarrow$$

$$V_{h_1,h_2,h_3} = t^{h_{\rm tot}-1}(P_{12} + P_{23} + P_{31})$$
$$\times\ (\text{Polynom. of order } h_{\rm tot} - 1)(P_{12}, P_{23}, P_{31})$$
$$M_{h_1,h_2,h_3} = t^{h_{\rm tot}} \times (\text{Polynom. of order } h_{\rm tot})(P_{12}, P_{23}, P_{31})$$

Geometric meaning: bulk lightcones

- In Minkowski, the hyperplanes $l \cdot x =$ const are the lightcones of points on a lightray at infinity
- Same in AdS with $l^\mu \partial_\mu z = 0$
- In the new lightcone frames with $l^\mu \partial_\mu z \neq 0$, the "hyperplanes" become lightcones of points on an ordinary bulk lightray
- The name "lightcone formalism" is finally true!

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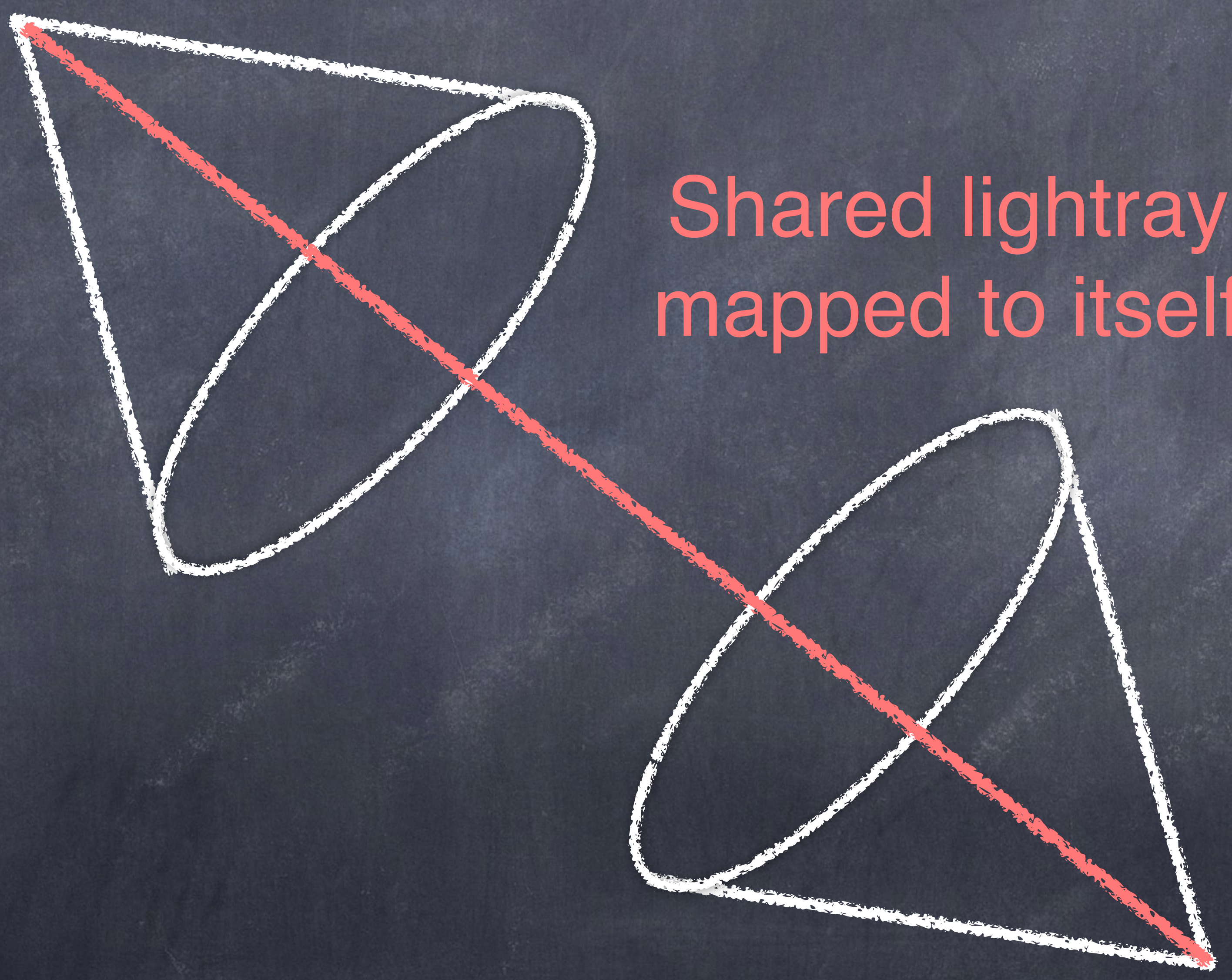
Causality in transition between lightcone frames



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I can’t do this without a blackboard

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Summary

- We extended the AdS lightcone formalism for massless fields with cubic interactions to bulk lightcones, and to de Sitter space
- The bulk lightcones allow to formulate & check causality properties
- Causality justifies the calculation of "scattering" in the de Sitter static patch
- The present formalism is a convenient tool for this calculation

Outlook

- Compute 3-point static-patch scattering for all spins
- Find patterns that match the structure of boundary CFT (Higher-Spin Algebra)
- Continue rewriting Metsaev:
  simplify the cubic-vertex polynomials
- Prove higher-order consistency of cubic chiral vertices in AdS (known in Minkowski)

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