The Festina Lente Bound (and some applications)

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1. Motivation
2. Swampland?
3. dS space, strings, swampland
4. A new hope?
5. The FL Bound
6. Pheno applications
7. Conclusions

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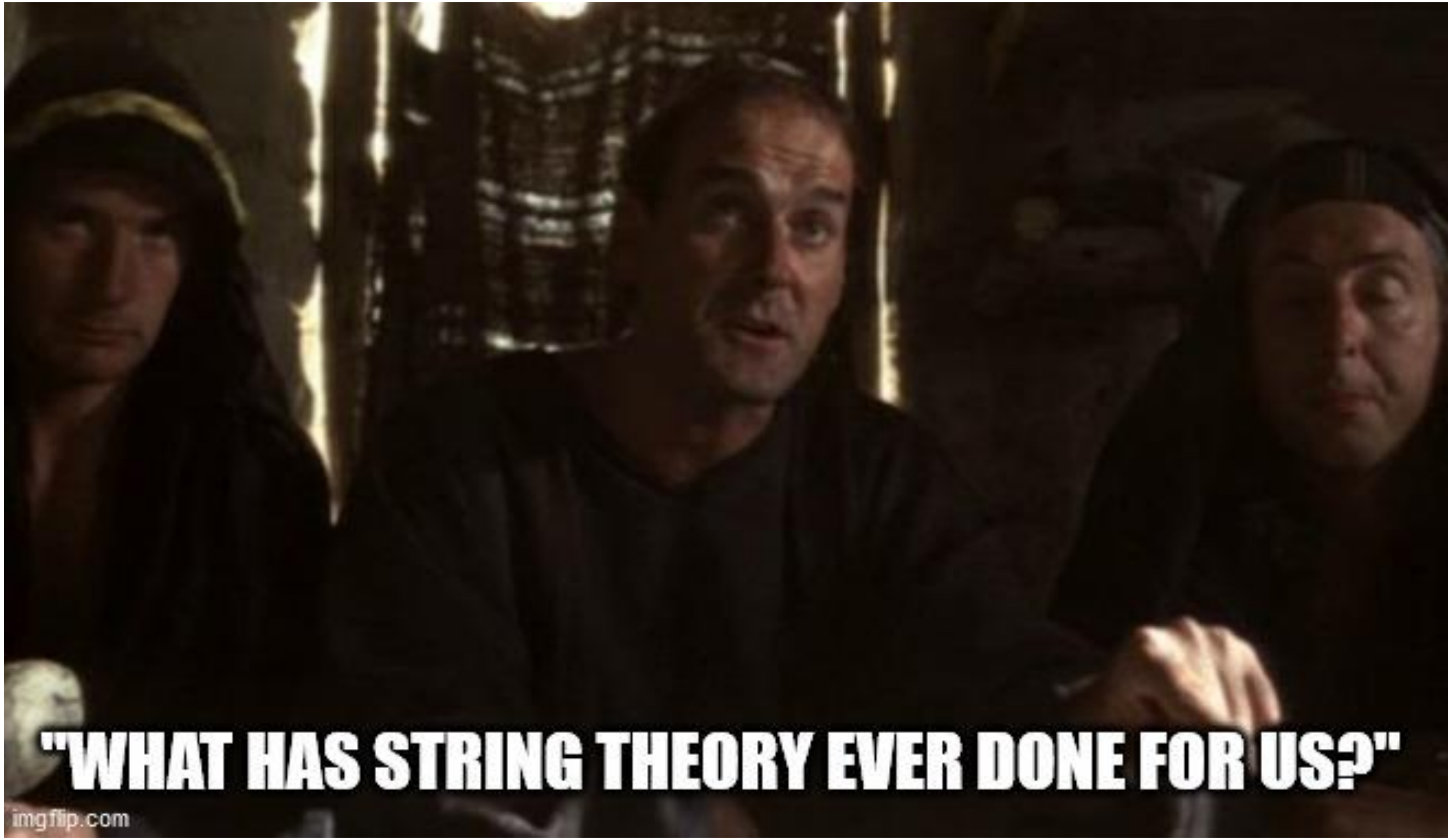
"WHAT HAS STRING THEORY EVER DONE FOR US?"
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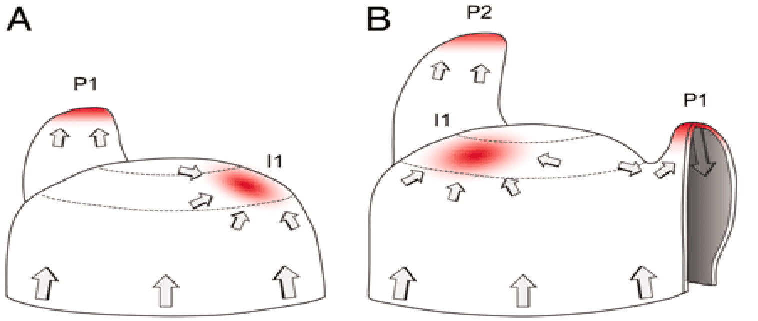
Illustration in biology

The growth of leaves on a plant: helical structure upwards , average angle between leaves is golden angle. Distribution of seeds on a flower

Why?

Answer 1: Leave distribution maximizes sunlight and rain absorption (Thompson, 1917)! Seed distribution maximizes amount of seeds on a given surface! Survival principle (Darwin).

Answer 2: It is the only option. Microscopic models for (Van der Waals) forces on lumps of cells during growth gives golden angle (Douady & Couder, 1992).



(personal interpretation of chapter from *Mathematics of Life*, from Ian Stewart, 2011)

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LANDSCAPE

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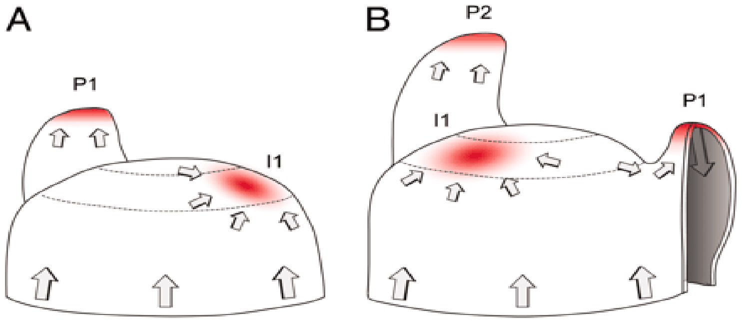
SWAMPLAND



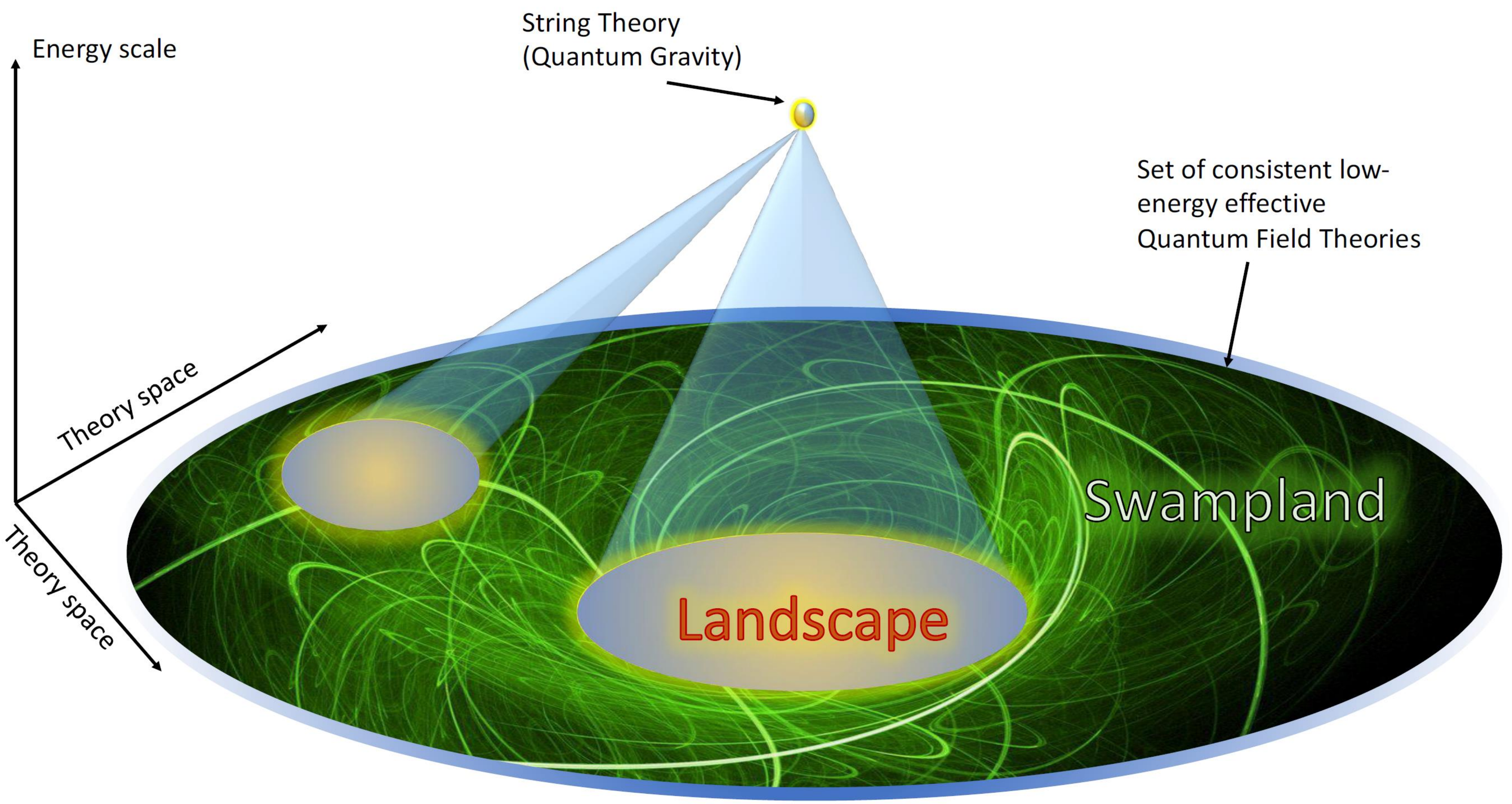
(personal interpretation of chapter from *Mathematics of Life*, from Ian Stewart, 2011)

The Swampland: Set of effective field theories **coupled to gravity** that cannot be UV completed”.

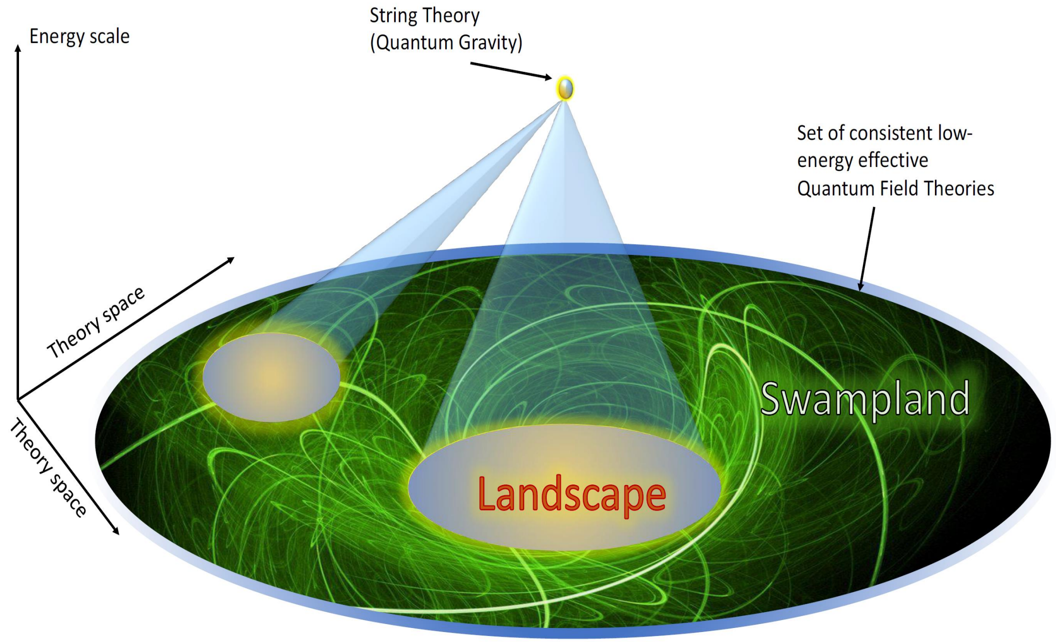
The landscape: the complementary set

Landscape: which effective field theories (EFTs) can we get from string theory (quantum gravity)?
Swampland: which EFTs can we **not** get?

Logically identical questions, “psychologically” different



Taken from E. Palti 1903.06239



Instead of trying to “reverse engineer” effective field theories and arrive at an “almost anything goes” picture (landscape), we ask: ‘what is not allowed?’.

Approach entirely different: *inequalities instead of equalities.*

- Keywords: **interdisciplinary** (pheno meets black hole physics, holography,…), **focusing on the ‘why’,** trying to find **patterns**.

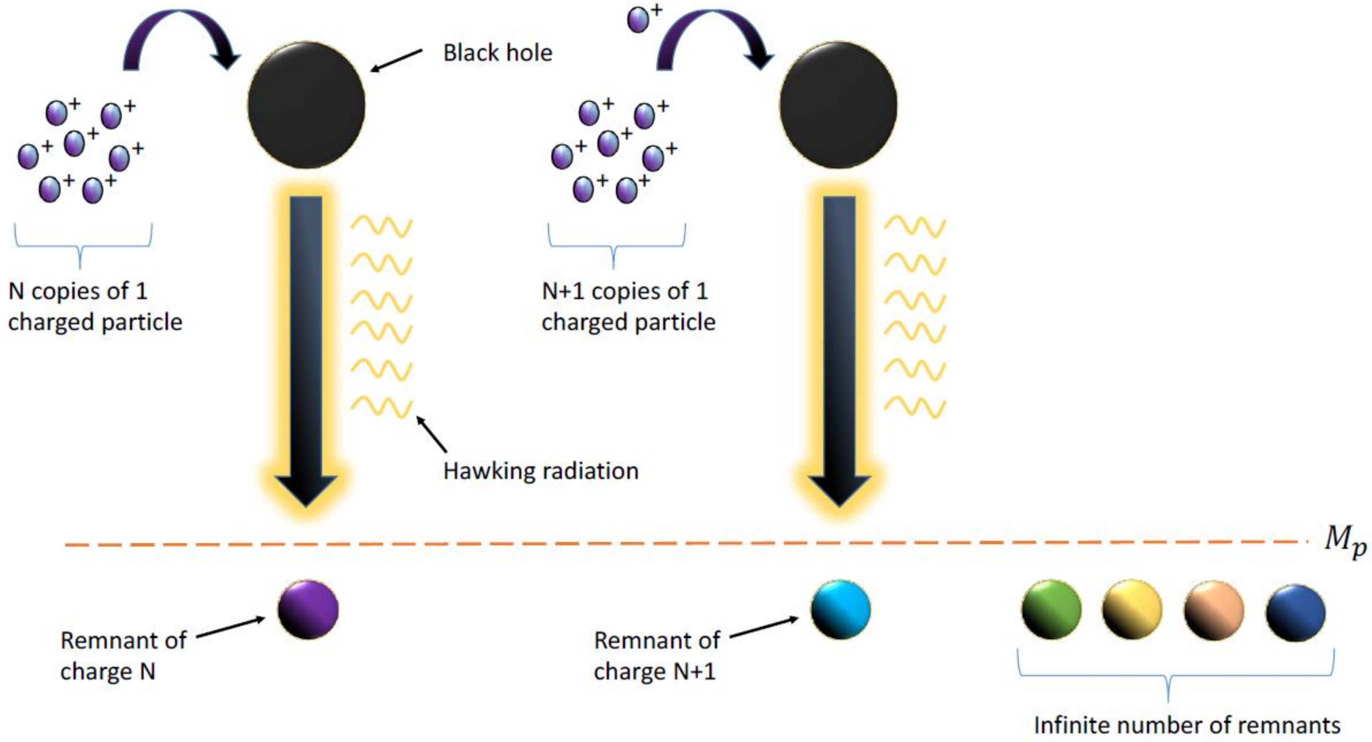
- *Conjectures* instead of *statements*. Become theorems when proven. Usually conjectures come from 1) patterns in string compactifications + 2) heuristic reasoning with black holes.

No global symmetries conjecture

Consider a field theory with a global symmetry that is not a gauge symmetry. This global symmetry will be broken when coupled to gravity. [Banks-Dixon 1988] [Harlow-Ooguri 2018])

- Indeed, every consistent compacti-fication of string theory has given field theories obeying this. Could have regarded this as circumstantial evidence.

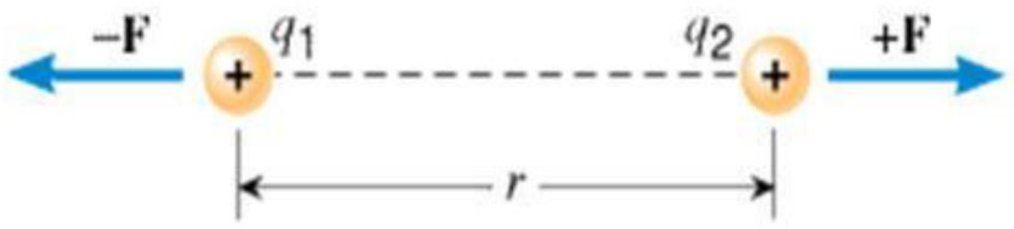
- Before the proofs, there were already heuristic black hole arguments.



OK, but it perhaps implies that gauge coupling constant cannot be arbitrary small? Gravity as weakest force?

Weak Gravity Conjecture [Arkani-Hamed, Motl, Nicolis, Vafa 2006]

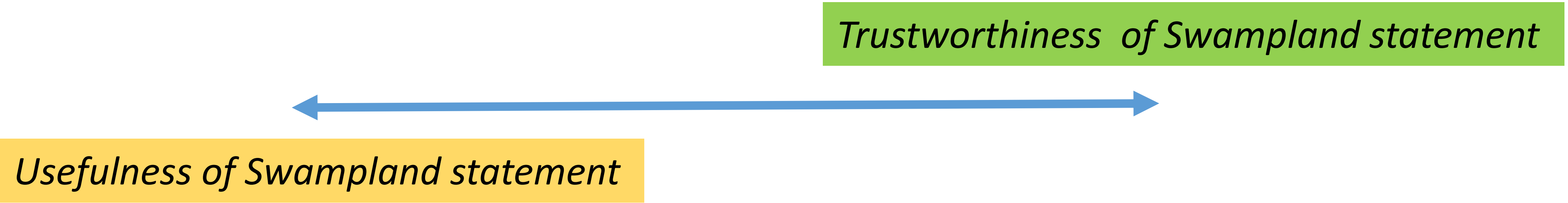
$$\frac{gq}{m} \geq \sqrt{\frac{d-3}{d-2}} M_p^{-\frac{(d-2)}{2}} \quad \text{for some charged state}$$



(b)

Constants in Nature not arbitrary, some parts of field theory space are empty when coupled to gravity, despite being "ok" (renormalisable, unitary…)

Current difficulty with Swampland program



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de Sitter space from strings?

UV completeness of string theory implies we know in principle how to compute vacuum energy, no cut off needed. But how? → Using the UV dof; *extra dimensions, branes, fluxes,….*

String theory in its usual form has 10 space-time dimensions & we "curl up" 6 of them:

Curvature gives 4D cc

$$ds_{10}^2 = ds_4^2 + ds_6^2$$

Metric on compact space. Finite size.

→ Associated length scale is called Kaluza-Klein scale (KK scale):

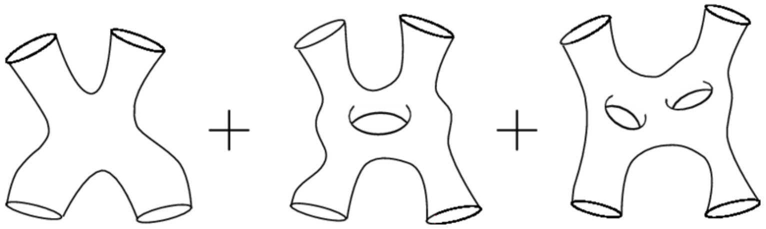
$$L_{KK} = \text{Volume}^{1/6} = \frac{1}{M_{KK}}$$

Mass scale associated with fluctuations of fields inside extra dimensions.

Most used approach to compute cc : construct a vacuum at the ***boundary of string moduli space.***

String theory reduces to classical 10D SUGRA if

1) g_s is small ($g_s << 1$):



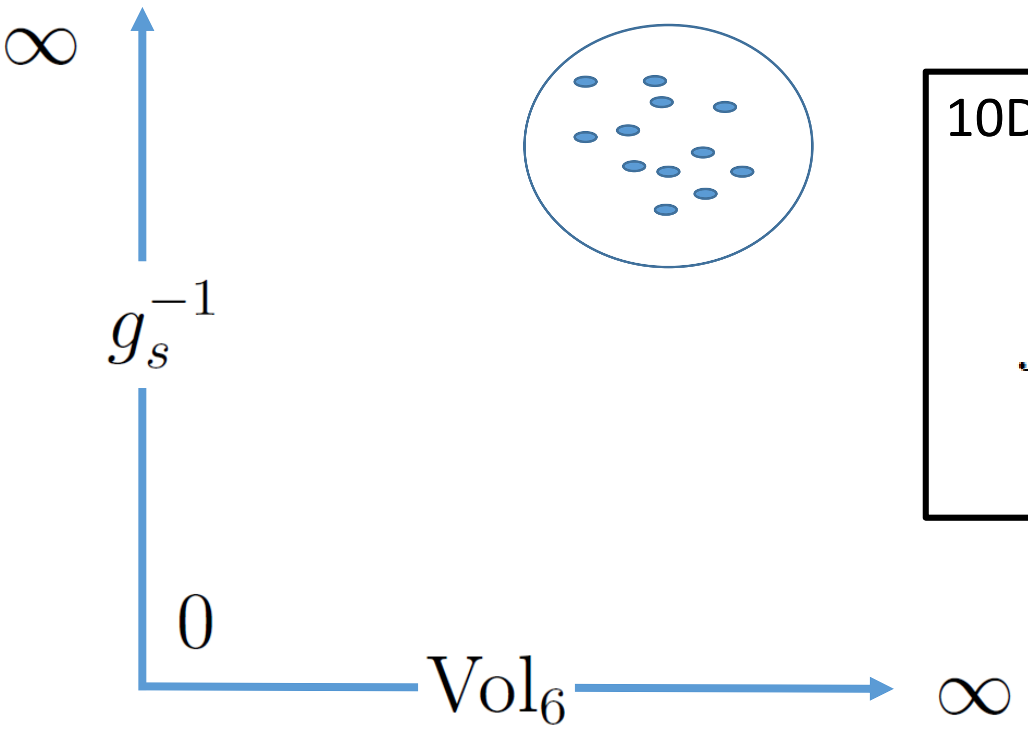
2) All field gradients are small with respect to $1/l_s$ to control higher derivative expansion. OK, if "curvature is small enough → volumes are large enough".

- Both g_s and volume *are fields in 4D* that should be stabilized in the vacuum!

- Note that :

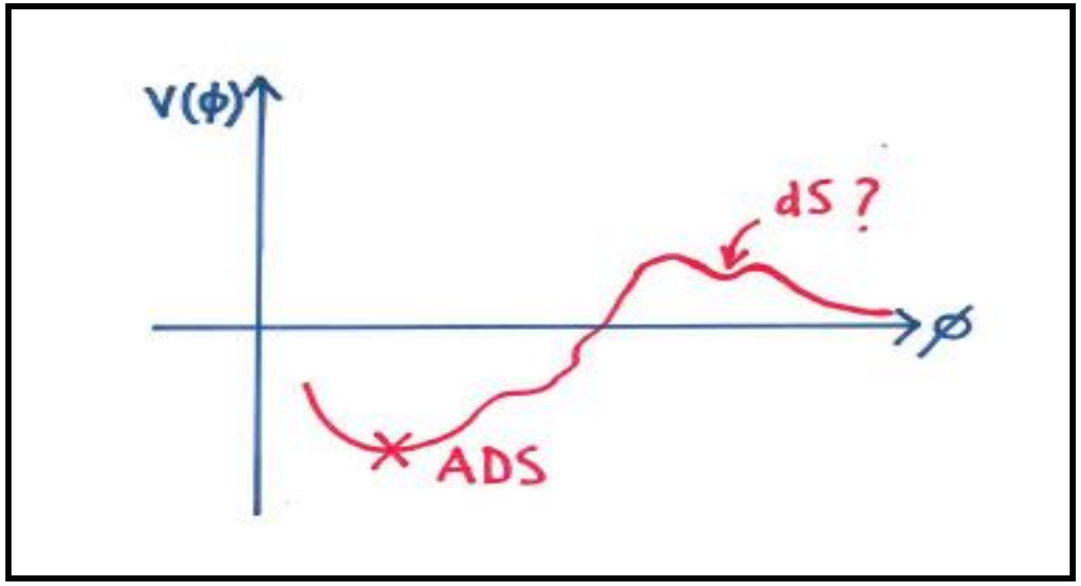
$$M_{pl}^2 = \frac{Vol_6}{g_s^2}\frac{1}{l_s^2}\,.$$

boundary of string moduli space:



10D sugra, possibly with some leading quantum corrections

$$\int \sqrt{g}\Big\{R - \tfrac{1}{2}(\partial\phi)^2 - \sum_n \frac{1}{2\,n!} e^{a_n \phi} F_n^2\Big\} + S_{loc}\,,$$



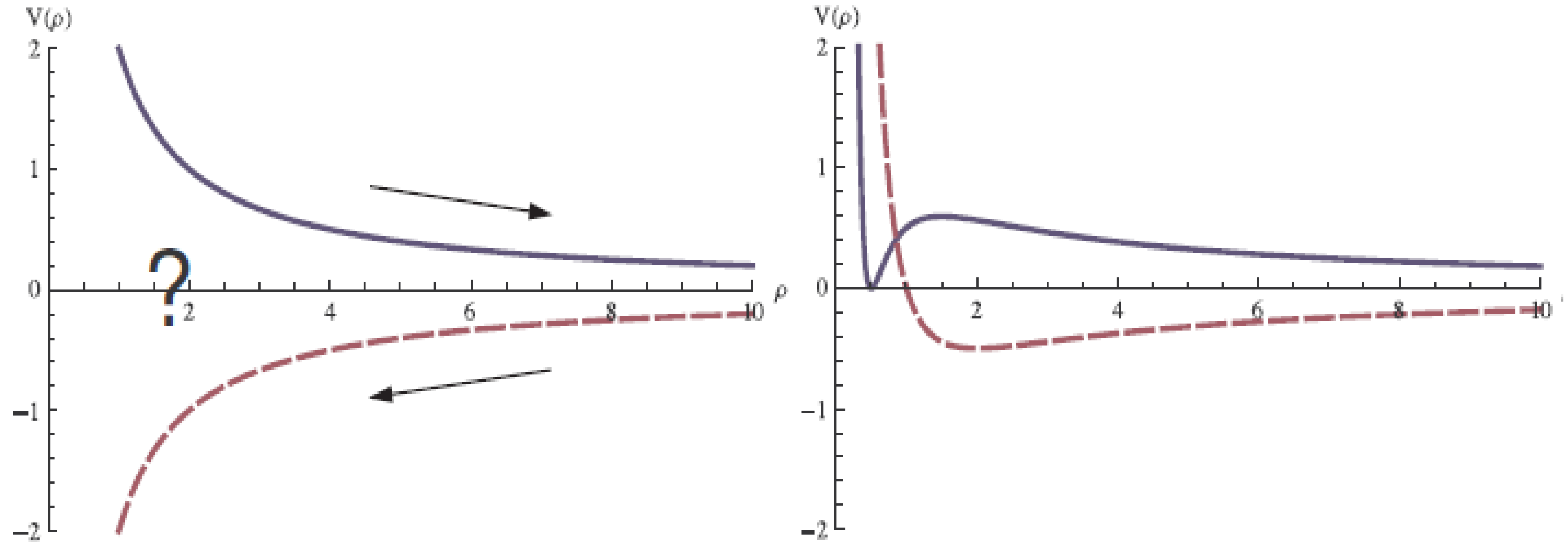
Vacuum energy:

E = Fluxes + Branes + Curvature

'Arrange' solutions such that quantum corrections are negligible or not.

Then the computed result is the full result (up to small corrections.) Nice virtue of string theory. We can compute vacuum energies in certain corners of the theory!

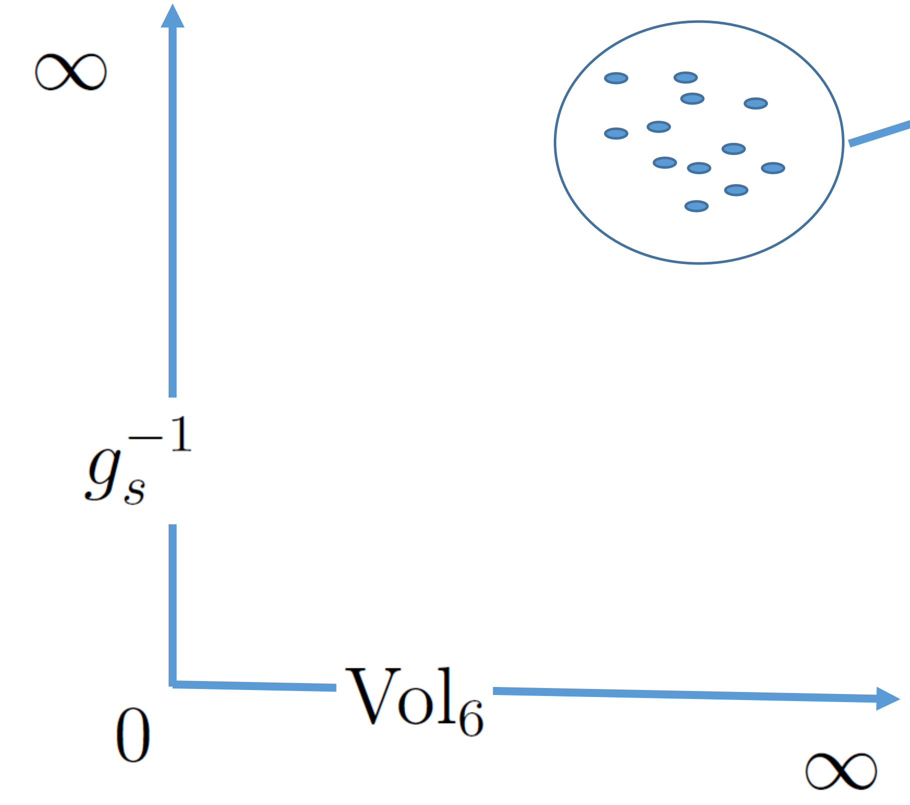
Fluxes are a way out of Dine-Seiberg problem: vacua are typically "non-calculable" [Denef review 2008]



Aim of flux compactification program is to construct ***calculable vacua.*** Solutions "under control". We can stabilize at the boundary of moduli space?

Recent developments have crushed this hope

boundary of string moduli space:



Only anti-de Sitter space here.

- Example $AdS_5 \times S_5$. As you crank up flux to infinity all length scales go to infinity, coupling is free parameter and can be dialed small. We trust it.
- Such a ``cranking up" never gives dS solutions. So no number that can be dialed. [Junghans 2018, Banlaki-Showdury-Roupec-Wrase, 2018]
- Consistent with heuristic (and more general) Swampland arguments. [Ooguri-Palti-Shiu-Vafa 2018, Wrase-Hebecker 2018]

De Sitter conjectures.

There are no dS solutions in the **parametric** controllable regimes [Palti, Shiu, Ooguri, Vafa 2018].

If there is a dS landscape, the vacua are at best meta-stable and not parametrically long lived.

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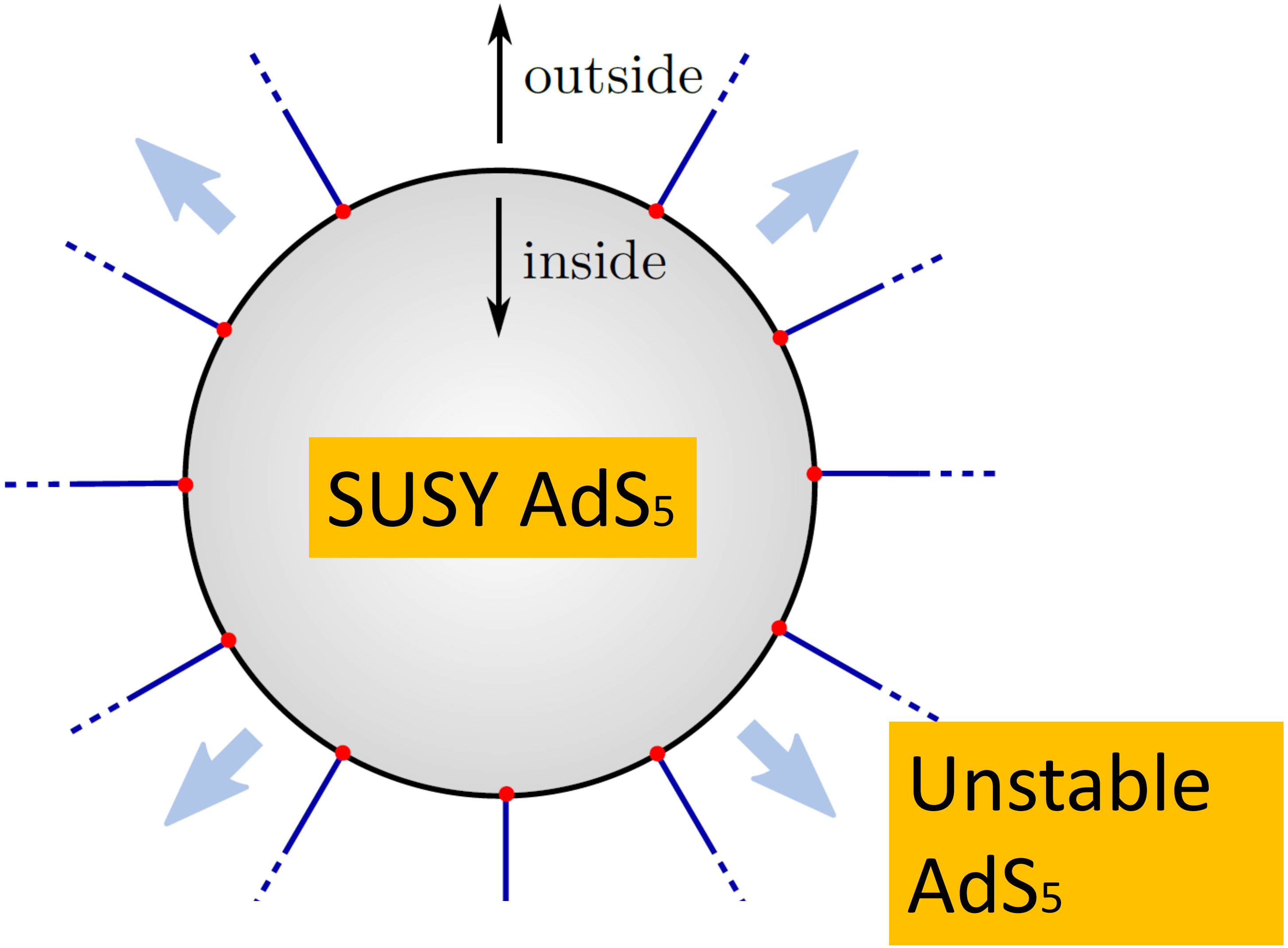
Idea Uppsala group [Banerjee, Danielsson, Dibitetto, Giri, Schillo 2018 & many follow-ups]:

There is a natural string theory embedding of brane worlds with de Sitter geometry exactly inspired by the Swampland logic!



$$\kappa_5 = 8\pi G_5 .$$

Denote the cc of the true and false vacuum as

$$\Lambda_\pm = -6k_\pm^2 \qquad \Lambda_- < \Lambda_+ < 0.$$

Shell metric: $ds^2_{\text{shell}} = -d\tau^2 + a(\tau)^2 d\Omega_3^2$

The Israel Junction condition gives the cosmological dynamics:

$$\sigma = \frac{3}{\kappa_5}\left(\sqrt{k_-^2 + \frac{1+\dot{a}^2}{a^2}} - \sqrt{k_+^2 + \frac{1+\dot{a}^2}{a^2}}\right) \Rightarrow \boxed{\dot{a}^2 = -1 + \frac{a^2}{R^2}}$$

Where brane tension is σ &

$$\kappa_4 = \frac{2k_-k_+}{k_- - k_+}\kappa_5 .$$

Physical picture

In the limit of large enough k the vacuum energy takes a simple expression

$$\rho_{\Lambda_4} \equiv \frac{3}{\kappa_4} R^{-2} = \frac{3\left(k_- - k_+\right)}{\kappa_5} - \sigma.$$

A bubble can only nucleate if its tension is smaller than $\sigma_{cr} = \frac{3}{k_5}(k_- - k_+)$

→ exactly the condition for having positive vacuum energy on the wall!

- It is a **toy model**, but Danielsson et al are expanding on this, including matter.
- *Explicit dS construction in string theory*? Maybe! Main difficulty; find an unstable AdS_5 that decays primarily through Coleman de Luccia bubbles. [Basile, Lanza 2020]

I now explain why this model-being UV complete-(almost) resolves the Big Bang singularity and the boundary choice problem [arXiv:2105.03253].

Big Bang?

From 5D viewpoint nothing is singular bout a=0 region. It is just non-existent. Bubble nucleates at finite radius. “What happens near a=0 is not a question that can or should be asked.”

Model is not past complete though, there is still some initial condition problem. Why was there an unstable AdS_5 to begin with? What created it?

Boundary condition problem

Is obvious now, the physics is that of decay through bubble nucleation (CDL). Natural expectation is tunneling wave function. We verified this is correct by checking that **CDL amplitude is Vilenkin's amplitude**;

CDL in 5D :

$$P = e^{-B} \qquad B = \frac{24\pi^2}{\kappa_4}\int_0^R da\sqrt{a^2 - \frac{a^4}{R^2}} = \frac{8\pi^2 R^2}{\kappa_4}$$

We verified further by using the expressions of Brown-Teitelboim

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Consider Einstein-Maxwell theory

$$S = \int d^d x \sqrt{-g} \left[\frac{1}{2} M_p^{d-2} \mathcal{R} - \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} - V \right].$$

For constant V, the Hubble radius is then fixed by

$$\frac{(d-1)(d-2)}{2\ell_d^2} = M_p^{2-d} V.$$

- The Electric Weak Gravity bound is: $$\frac{gq}{m} \geq \sqrt{\frac{d-3}{d-2}} M_p^{-\frac{(d-2)}{2}} \quad \text{for some charged state}$$

- The Festina Lente bound is: $$m^4 \gtrsim (gq)^2 V \quad \text{for every charged state}$$

In 4D, in terms of fine structure constant, we have a window:

$$(8\pi\alpha V)^{1/4} < m < (8\pi\alpha)^{1/2} M_{\mathrm{P}}$$

$$\alpha = \frac{g^2}{4\pi}$$

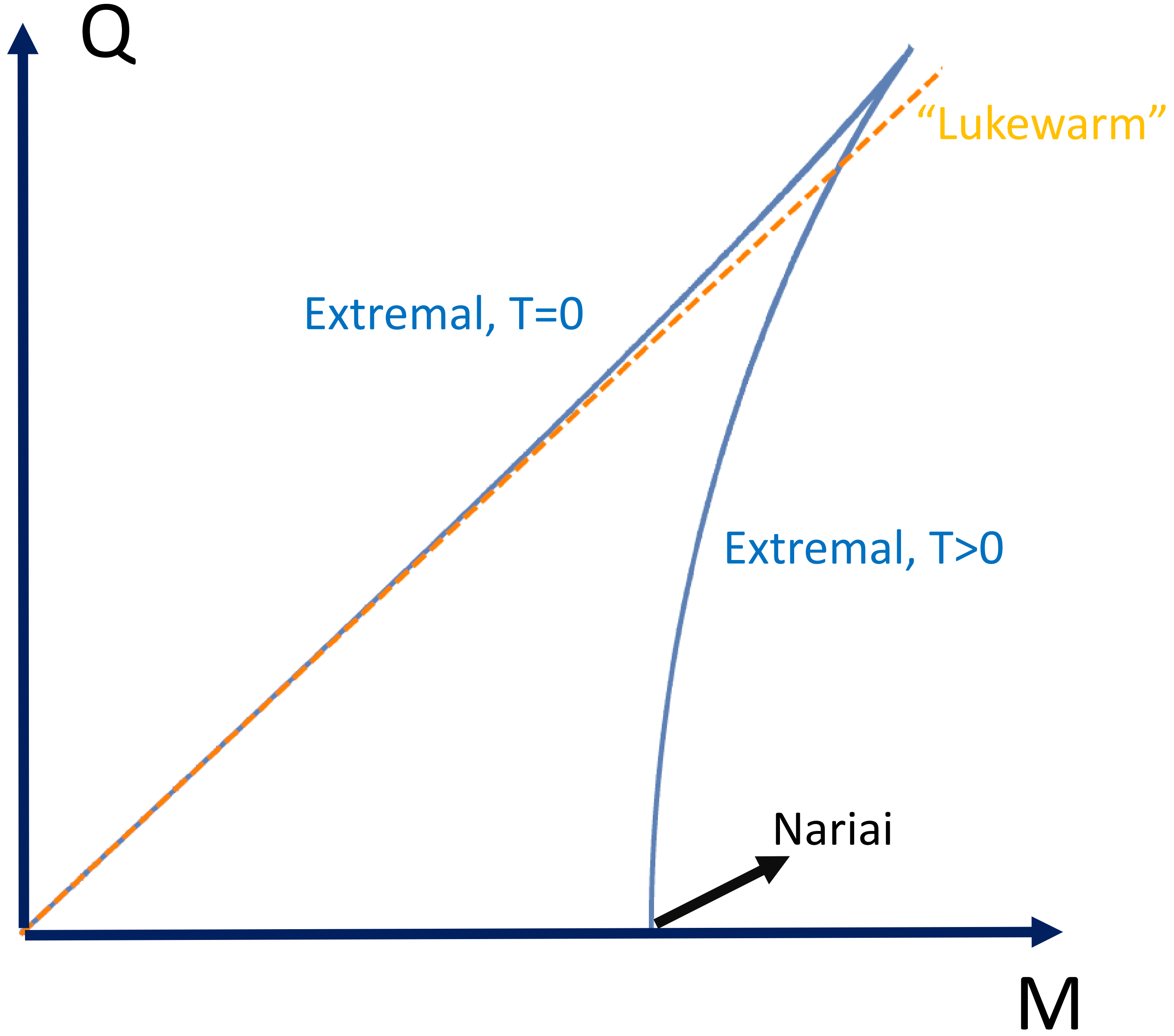
Argument 1: Quantum dynamics of charged black holes in de Sitter space

$$ds^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} + r^2 d\Omega,$$

$$U(r) \equiv 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - r^2$$

$$M \equiv \frac{GM}{\ell}, \quad Q^2 \equiv \frac{Gg^2Q_r^2}{4\pi\ell^2}.$$

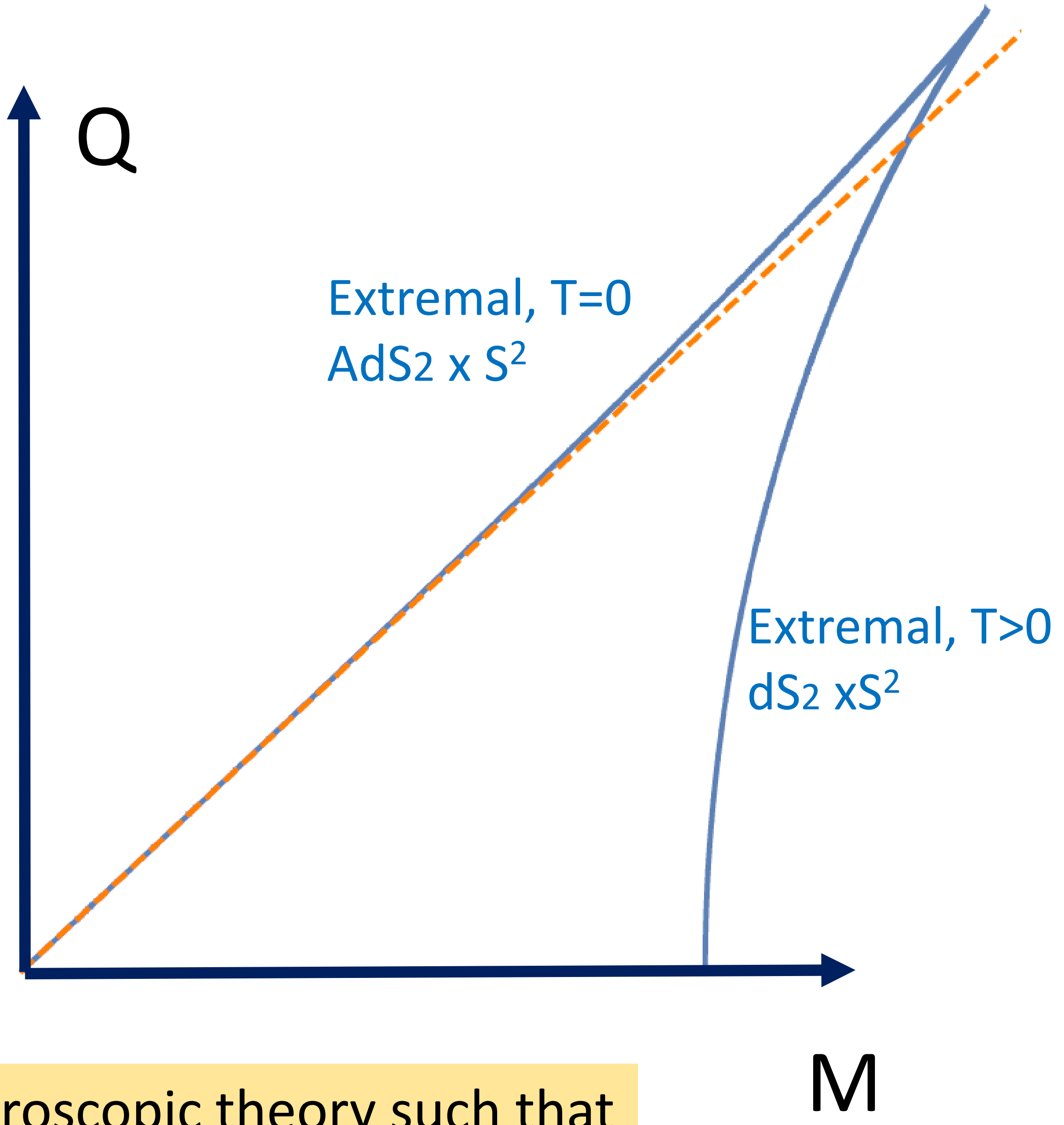
$$S = \frac{\pi}{4G}\left(r_{BH}^2 + r_{CH}^2\right)$$



Weak gravity principles for extremal black holes?

- **Left extremal branch**. Like in flat space. But now black holes unstable without even requiring weak gravity. → de Sitter expansion helps the Schwinger effect. Always unstable. Need time scales?

- **Right extremal branch. Charged Nariai.** Gigantic black holes probing cosmic horizon. Super-extremal if black hole horizon catches up with cosmic horizon. Should be forbidden = *Cosmic censorship*.

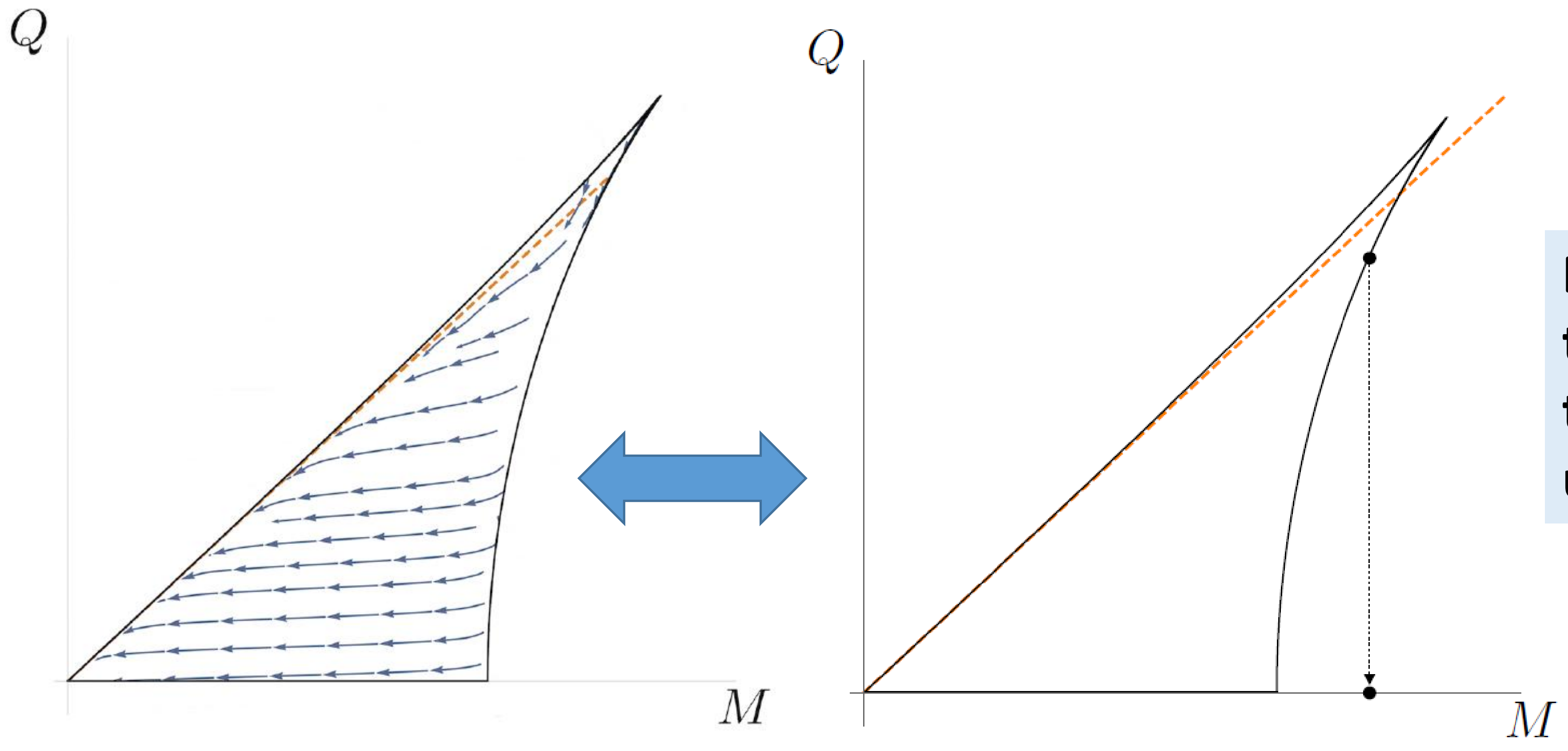


Guiding principle: constrain microscopic theory such that black holes *do not decay to the super-extremal side*.

Adiabatic motion in Q,M plane. Semi-classical analysis of Hawking&Schwinger radiation:

$$\dot{Q} = -4\pi\mathcal{J}, \qquad \frac{4r(r\dot{M} - Q\dot{Q})}{-2Mr + Q^2 - r^4 + r^2} = -16\pi r^4 G\mathcal{T}.$$

[Montero & Venken & VR 2019 , Lüben& Lüst & Ribes Metidieri 2020]



Details J and T are such that evolution brings you to super-extremal branch unless you obey FL bound.

Argument 2: Magnetic Weak Gravity & Completeness

The magnetic WGC: $$\Lambda_{EFT} \leq g M_P$$

Can be found from demanding that a monopole of unit charge is larger than its corresponding black hole solution. In dS space we must also demand that the monopole is smaller than the charged Nariai solution, ie, it fits in dS space [Huang & Li &Son 2006].

This leads to $$g^2 \geq \frac{3}{2}\left(\frac{H}{M_P}\right)^2$$ (This is saying dark energy scale is below cut-off)

We get logical triangle with Festina Lente; applying FL to the (electric) WGC particle :

$$m \lesssim g M_P \quad \text{and} \quad m^2 \gtrsim g M_P H \quad \Rightarrow \quad g \gtrsim \frac{H}{M_P},$$

This allows us to fix the unknown constant in FL bound

$$m^2 \geq \sqrt{6}\, g M_P H.$$

Note how the inequality

$$g^2 \geq \frac{3}{2}\left(\frac{H}{M_P}\right)^2$$

Resonates with the Swampland bounds that forbid dS vacua at parametric weak coupling!

Even when you are a Swampland critic, you surely appreciate the inner consistency of this all!

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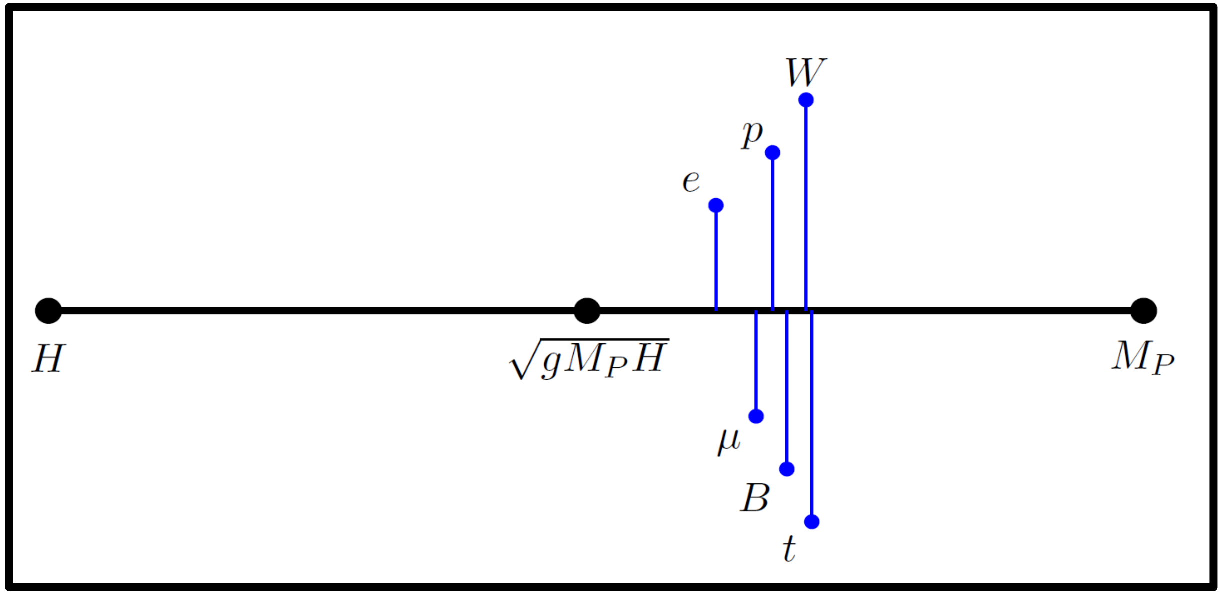
- All charged fields in the SM obey FL ☺
- Can FL help with explaining hierarchy problems?

→ CC hierarchy (Planck units): $$\Lambda \lesssim \frac{m^4}{4\pi\alpha}\,; \quad \xrightarrow{\text{Electron}} \quad \Lambda \lesssim 3 \cdot 10^{-89}\,;$$

→ Electro weak hierarchy: $$v^2 \gtrsim \frac{1}{g} M_P H = \frac{V^{1/2}}{g}$$ (W-bosons, g=SU2 coupling)



Logarithmic scale

A non-abelian gauge theory automatically contains massless charged states: the gluons. Nariai black holes? → Use the Cartan of the gauge group. So massless non-abelian gauge fields are in contradiction with FL.

→There cannot be a phase of the Standard Model where the weak interaction is long range → no local minimum at Phi = 0 for the Higgs potential.

→The other possibility consistent with non-abelian gauge fields and FL is confinement. Is realized by the gluons in the SM.

FL predicts that in a de Sitter background non-abelian gauge fields must confine or be spontaneously broken, at a scale above H.

$$m_{\text{Gauge field}} \gtrsim H, \quad \text{or} \quad \Lambda_{\text{Confinement}} \gtrsim H$$

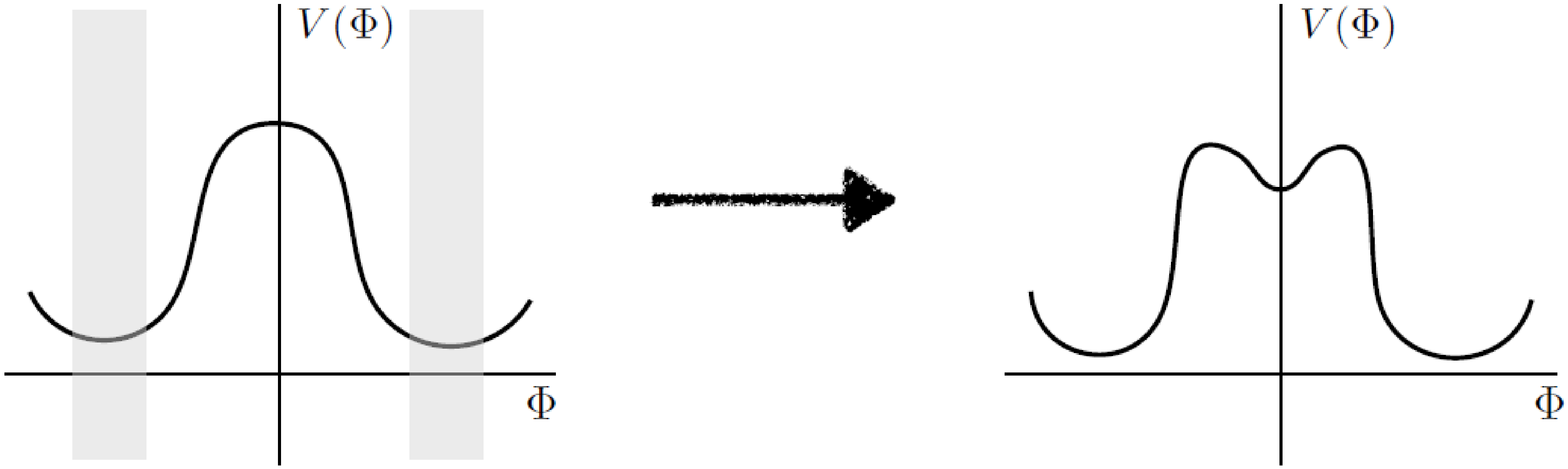


Figure 2. On the left, we show the usual shape of the "Mexican hat" Higgs potential, which arises from equation (4.5). However, only the region shaded in gray has been accessed experimentally. It is conceivable that the region near $\Phi \approx 0$ has a different shape, for instance, that of the "cowboy hat"

Neutrino's?

- Suggestive numerology $\sqrt{gM_P H} \sim 10^{-3}\, eV$
- If B-L is weakly gauged instead of spontaneously broken at high E, then lightest neutrino cannot be massless.

Swampland relations with top down models of dS?

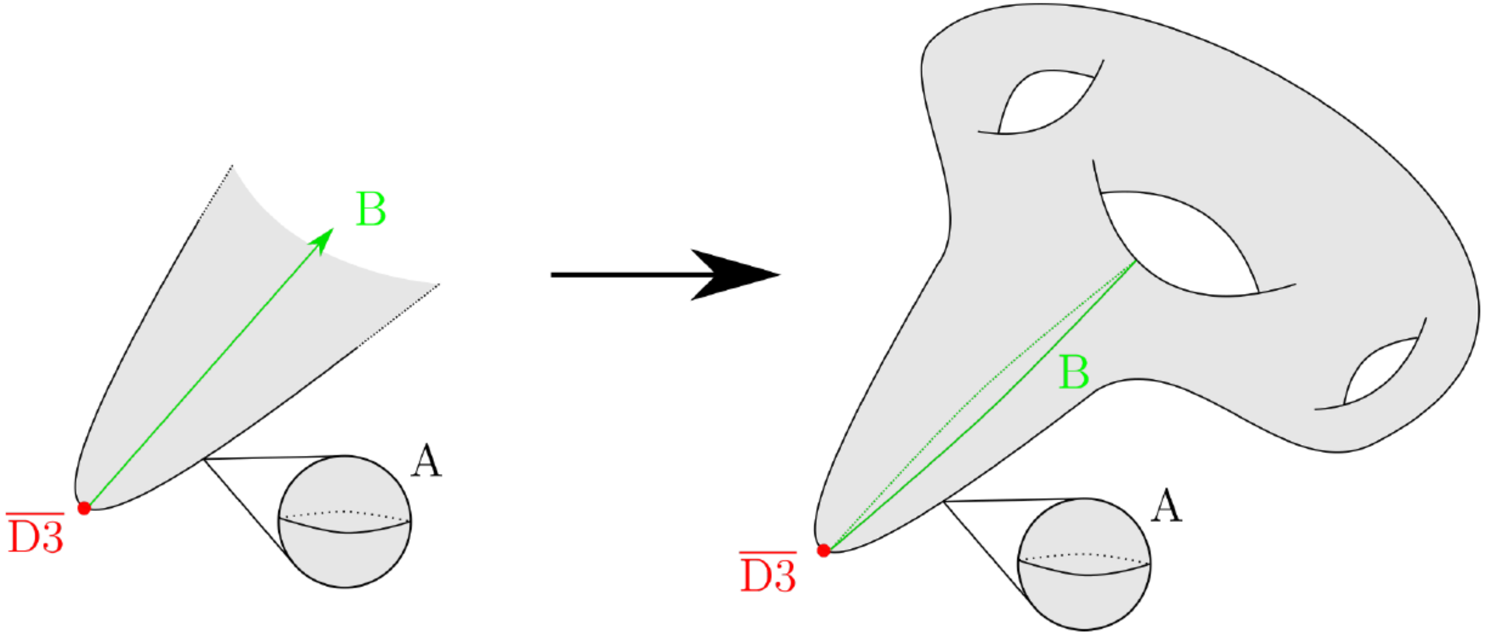
FL, like the WGC or other Swampland bounds can inform us about the control & consistency of a compactification. Provided you trust the Swampland bounds. But WGC is rather well established. FL is similar in spirit….

For instance dS model building including non-Abelian gauge fields without confinement or Higgsing should be impossible…

→ Lets apply to KKLT.

Amazingly, FL bounds and WGC **bounds correspond exactly to control bounds** for anti-brane-uplifting in KKLT & LVS [Montero-Vafa-Venken-VR 2021].

$$ds_{10}^2 = e^{2A_0}\sigma^{1/2}g_4 + r_0^2\left[\frac{1}{2}d\tau^2 + d\Omega_3^2 + \frac{\tau^2}{4}d\Omega_2^2\right], \qquad e^{2A_0} = r_0^{-2}e^{-4\pi\frac{K}{3g_sM}}.$$

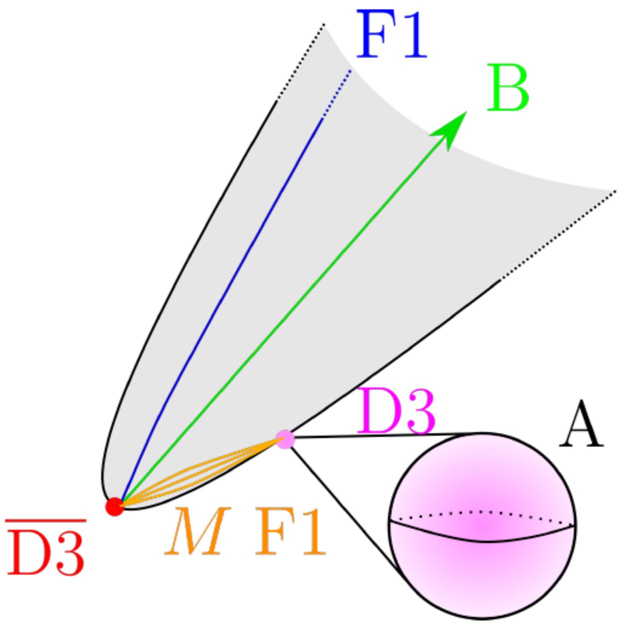


$$r_0 = \sqrt{g_sM}\,l_s\,.$$

Figure 4. The KS throat and its compactification.

In Planck units: $V_{\overline{D3}} \sim \frac{g_s^3}{4\pi} \frac{e^{4A_0}}{\sigma_0^2}$

$$V = \alpha V_{\overline{D3}}\,, \quad \rightarrow \quad H M_p = \sqrt{\frac{\alpha \sigma_0}{g_s}} e^{2A_0} \qquad 0 < \alpha < 1,$$ (no finetuning)



- Strings attached to anti-D3 and bulk branes.
- D3 wrapped around 3-cycle.
- ….

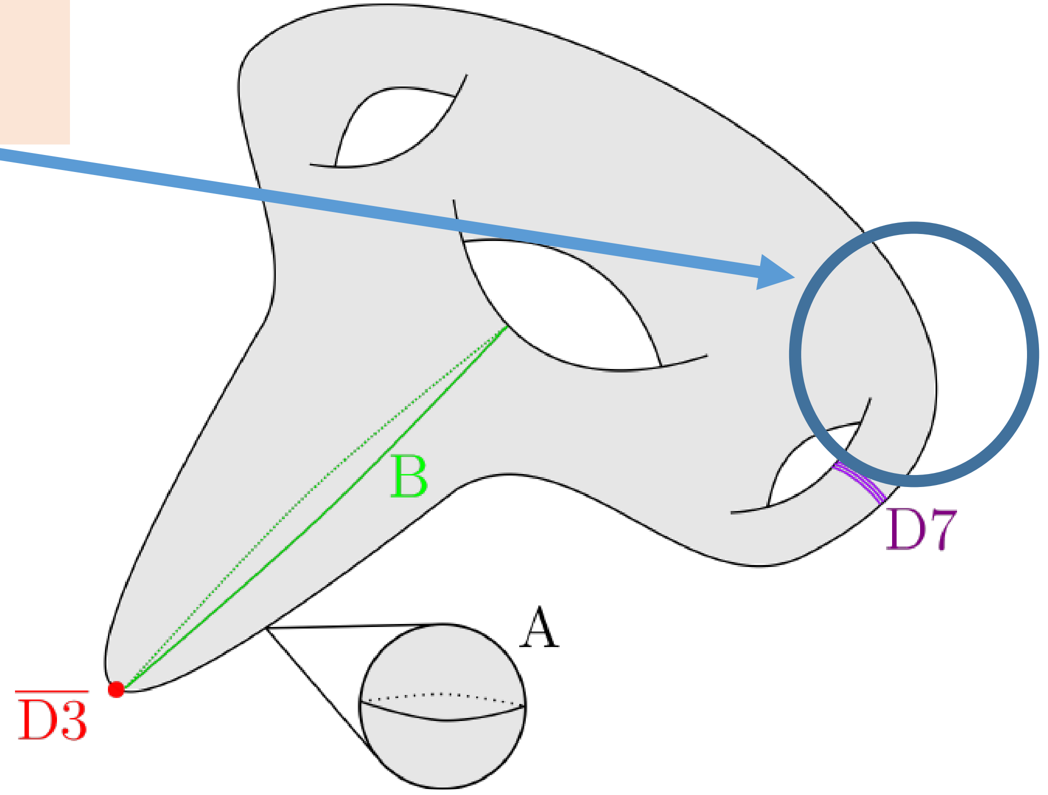
WGC and FL inequalities ALL reduce to KNOWN stability and control requirements for KKLT anti-brane uplifting.

- *FL on D3 particle:* $r_0^4 > \sqrt{4\pi\alpha} g_s$
- *FL for string excitations:* $(r_0)^2 >> \sqrt{\alpha}\,,$
- *WGC on D3 particle:* $e^{-\frac{2\pi K}{3 g_s M}} < \sqrt{\frac{4\pi\sigma}{g_s}}\,.$

These are <u>local</u> stability & control conditions.

BUT

The (necessary?) requirement of decoupling throat from bulk does allow one to engineer models that violate FL!



→ Resonates well with the "anti-brane debates" of the last 4 years.

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Swampland & de Sitter

- The Swampland paradigm is the necessary wind of change for string phenomenology.
- The current tension is the inverse relation between trustworthiness and usefulness for pheno.
- The fate of dS is unclear. But Swampland logic implies it is not there in regions of parametric control!
- I presented a crispy new idea for constructing dS space on bubbling brane worlds. It shows how debates in quantum cosmology can be settled if you have UV completion!
- Once we *assume* dS is possible, we can apply Swampland logic to matter content → FESTINA LENTE.

The FL bound

- The FL bound is a non-trivial UV IR connection “derived” on the basis of similar principles to WGC. It connects to other bounds (magnetic WGC), demanding UV cut-off is above dark energy scale.

- It can constrain BSM models, but more work needs to be done! It “postdics” Higgsing and confinement.

- For top down model building it actually “postdics” many known consistency relations on anti-brane uplifting. But it implies decoupling bulk from throat is not possible!

Thank you!

Consider gravity coupled to a *positive* cosmological constant and an FLRW Ansatz:

$$ds^2 = -N^2(\tau)\mathrm{d}\tau^2 + a(\tau)^2\mathrm{d}\Omega_3^2$$

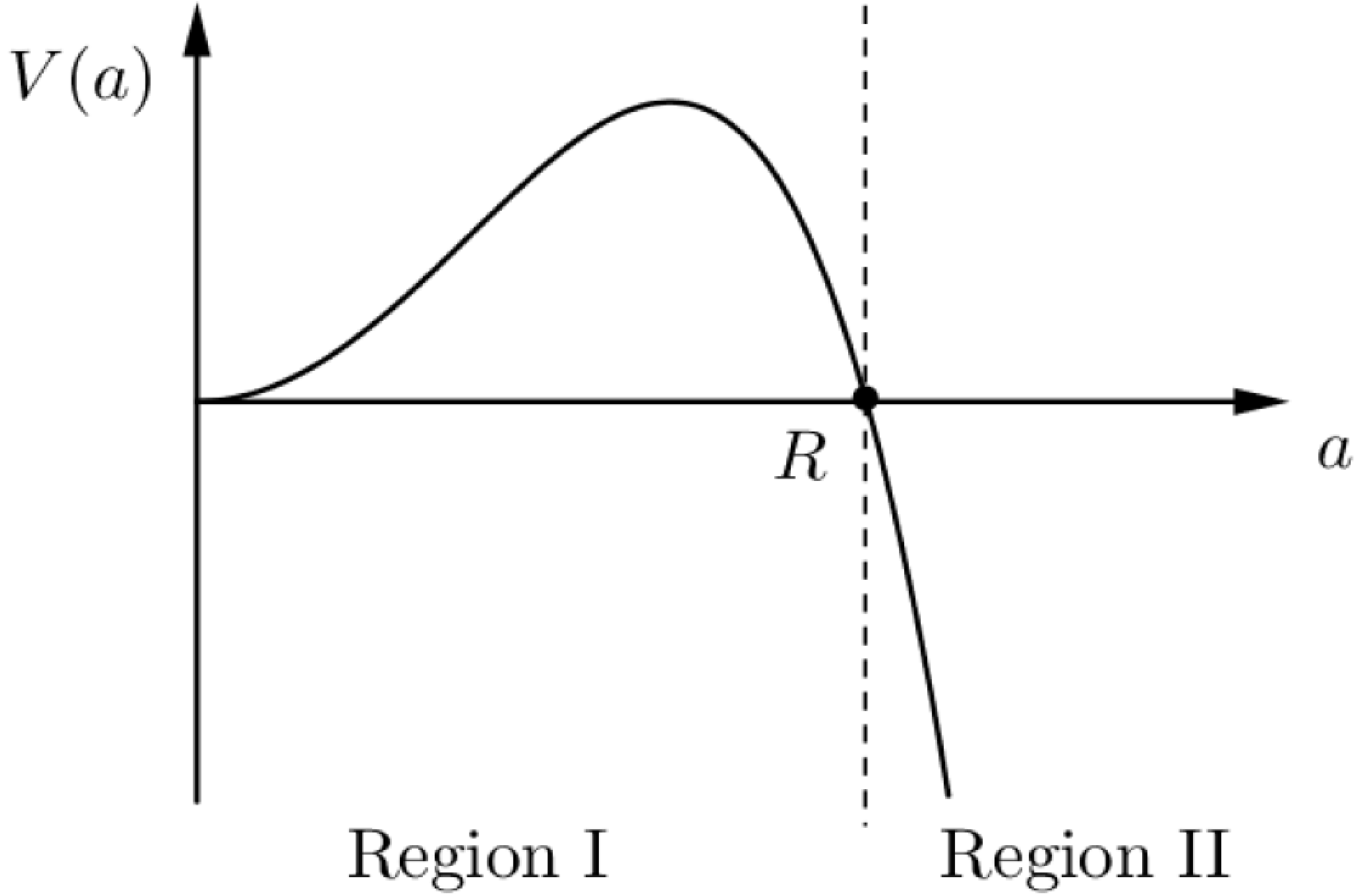
Friedman equation; $\dot{a}^2 = -1 + \frac{a^2}{R^2}$ with R the de Sitter length.

A quantisation of the effective one-dimensional action (mini-superspace)

$$S = \frac{6\pi^2}{\kappa_4}\int \mathrm{d}\tau N\left(-\frac{a\dot{a}^2}{N^2} + a - \frac{a^3}{R^2}\right)$$

gives the following Wheeler deWitt equation:

$$\frac{N}{a}\left(-\frac{1}{24\pi^2}\frac{d^2}{da^2} + 6\pi^2 V(a)\right)\Psi(a) = 0\,, \qquad V(a) = a^2 - \frac{a^4}{R^2}$$



$$S(a, a_i) \equiv \frac{12\pi^2}{\kappa_4} \int_{a_i}^{a} \sqrt{|V(a')|}da', \qquad S_0 \equiv S(R, 0) = \frac{4\pi^2 R^2}{\kappa_4}$$

For simplicity we discuss WKB solution

$$\Psi_{\rm I}(a) = \frac{1}{|V(a)|^{1/4}} \left(c e^{S(a,0)} + d e^{-S(a,0)}\right) ,$$
$$\Psi_{\rm II}(a) = \frac{1}{|V(a)|^{1/4}} \left(A e^{iS(a,R)} + B e^{-iS(a,R)}\right) ;$$

Connection formulae relate constants:

$$c = \tfrac{1}{2} e^{-S_0} \left(A e^{i\frac{\pi}{4}} + B e^{-i\frac{\pi}{4}}\right)$$
$$d = e^{S_0} \left(A e^{-i\frac{\pi}{4}} + B e^{i\frac{\pi}{4}}\right) ,$$

Normalisation is conventionally chosen such that: $\lim_{a\to 0} |V(a)|^{1/4} \Psi(a) = 1 \,.$

→ Still need to fix boundary conditions to pick a wavefunction uniquely.

$$S_0 \equiv S(R,0) = \frac{4\pi^2 R^2}{\kappa_4}$$

Hartle Hawking (no boundary) $(c,d) = (1,0)$

$$\Psi_{\rm HH}(a) = \frac{1}{|V(a)|^{1/4}} \begin{cases} e^{S(a,0)} & \text{Region I} \\ 2e^{S_0}\cos\left(S(a,R) - \frac{\pi}{4}\right) & \text{Region II} \end{cases} .$$

$$P_{\rm HH} \propto e^{2S_0} .$$

Vilenkin (tunneling) $(A,B) = (0,B)$

$$\Psi_{\rm V}(a) \approx \frac{1}{|V(a)|^{1/4}} \begin{cases} e^{S_0} e^{-S(a,0)+i\frac{\pi}{4}} & \text{Region I} \\ e^{-iS(a,R)} & \text{Region II} \end{cases} ,$$

$$P_{\rm V} \propto e^{-2S_0}$$