Conservation law for harmonic mappings in higher dimensions

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It has been a longstanding open problem to find a direct conservation law for harmonic mappings into manifolds. In the late 1980s, Chen and Shatah independently found a conservation law for weakly harmonic maps into spheres, which can be interpreted by Noether's theorem. This leads to Helein's celebrated regularity theorem on weakly harmonic maps from surfaces. For general target manifolds, Riviere discovered a direct conservation law in two dimension in 2007, allowing him to solve two well known conjectures of Hildebrandt and Heinz. As observed by Riviere-Struwe in 2008, due to lack of Wente's lemma, Riviere's approach does not extend to higher dimensions. In a recent joint work with Chang-Lin Xiang, we successfully found a conservation law, in the spirit of Riviere, for a class of weakly harmonic maps (around regular points) into general closed manifolds in higher dimensions.