

# FY2023 Annual Report

Gravity, Quantum Geometry and Field Theory Unit

Assistant Professor Reiko Toriumi

## Abstract

The Unit's research concerns quantum gravity, quantum geometry, quantum field theory. Within those frameworks, we study random matrices/tensors, matrix/tensor models, matrix/tensor field theories, QFT on (random) graphs, with additional feature such as causality and Lorentzian structures, and nonperturbative aspects such as resurgence and functional renormalisation group. We also consider related mathematical subjects such as knot theory, matroids, and free probability to expand and deepen the understanding of the central theme of research motivated by random discrete geometric path integral formulation of quantum gravity.

## 1. Staff

- Dr. Reiko Toriumi, Group Leader
- Dr. Remi Avohou, Staff Scientist
- Dr. Rudrajit Banerjee, Postdoc
- Dr. Nicolas Delporte, Postdoc
- Dr. Cihan Pazarbaşı, Postdoc
- Juan Abranches, Graduate Student
- Saswato Sen, Graduate Student
- Andreani Petrou, Graduate Student

## 2. Collaborations

### 2.1 From Polymatroids to Delta-polymatroids

- Description: Ongoing.
- Type of collaboration: Joint research
- Researchers:
  - Prof. Tamas Kalman, Tokyo Institute of Technology, Japan
  - Dr. Remi Avohou, OIST

### 2.2 Classifying Graphs for Higher Grades for Multi-orientable Multi-Matrix Model

- Description: On ArXiv.
  - Type of collaboration: Joint research
  - Researchers:
    - Prof. Reiko Toriumi, OIST
    - Matthias Van Craeynest, University of Edinburgh, UK
    - Dr. Remi Avohou, OIST
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## 2.3 On Gross, Mansour and Tucker Conjecture for $\Delta$ -matroids

- Description: On ArXiv.
- Type of collaboration: Solo research
- Researchers:
  - Dr. Remi Avohou, OIST

## 2.4 N-cutoff regularization for fields on hyperbolic space

- Description: Published.
- Type of collaboration: Joint research
- Researchers:
  - Dr. Rudrajit Banerjee, OIST
  - Dr. Maximilian Becker, Radboud University, the Netherlands
  - Dr. Renata Ferrero, FAU Erlangen, Germany

## 2.5 States of Low Energy on Bianchi I spacetimes

- Description: Published.
- Type of collaboration: Joint research
- Researchers:
  - Dr. Rudrajit Banerjee, OIST
  - Prof. Max Niedermaier, University of Pittsburgh, USA

## 2.6 A Wick rotated heat kernel for admissible complex metrics

- Description: Ongoing.
- Type of collaboration: Joint research
- Researchers:

- Dr. Rudrajit Banerjee, OIST
- Prof. Max Niedermaier, University of Pittsburgh, USA

## 2.7 Exact Quantization and Phase Transition

- Description: Ongoing.
- Type of collaboration: Joint research
- Researchers:
  - Dr. Cihan Pazarbaşı, OIST
  - Assoc. Prof. Tatsuhiro Misumi, Kindai University, Japan

## 2.8 Phase Transition from Perturbative Expansion

- Description: Ongoing.
- Type of collaboration: Solo Research
- Researchers:
  - Dr. Cihan Pazarbaşı, OIST

## 2.9 Tensor Eigenvalues and Free Probability

- Description: Ongoing.
- Type of collaboration: Joint research
- Researchers:
  - Prof. Benoit Collins, Kyoto University
  - Dr. Nicolas Delporte, OIST
  - Prof. Reiko Toriumi, OIST

## 2.10 The Edge of Random Tensor Eigenvalues with Deviation

- Description: On arxiv.
- Type of collaboration: Joint research
- Researchers:
  - Dr. Nicolas Delporte, OIST
  - Prof. Naoki Sasakura, Yukawa Institute for Theoretical Physics, Kyoto, Japan

## 2.11 Power Iteration for Random Tensors

- Description: Ongoing.

- Type of collaboration: Joint research
- Researchers:
  - Dr. Danielle Bielli, Chulalongkorn University, Thailand
  - Dr. Nicolas Delporte, OIST
  - Prof. Oleg Evnin, Chulalongkorn University, Thailand

## **2.12 Renormalization for Quantum Fields on Random Geometries**

- Description: Ongoing.
- Type of collaboration: Joint research
- Researchers:
  - Dr. Rudrajit Banerjee, OIST
  - Dr. Nicolas Delporte, OIST
  - Saswato Sen, OIST
  - Prof. Reiko Toriumi, OIST

## **2.13 Numerical Simulation of JT Gravity at Finite Cutoff**

- Description: Ongoing.
- Type of collaboration: Joint research
- Researchers:
  - Dr. Nicolas Delporte, OIST
  - Prof. Frank Ferrari, Université Libre de Bruxelles, Belgium

## **2.14 Harer-Zagier formulas for families of twisted hyperbolic knots**

- Description: Published.
- Type of collaboration: Joint research
- Researchers:
  - Prof. Shinobu Hikami, OIST
  - Prof. Reiko Toriumi, OIST
  - Andreani Petrou, OIST

## **2.15 Knot invariants from a matrix model perspective**

- Description: Ongoing.
- Type of collaboration: Joint research
- Researchers:
  - Prof. Shinobu Hikami, OIST

- Prof. Reiko Toriumi, OIST
- Andreani Petrou, OIST

## **2.16 Dually Weighted Multi-matrix Models as a Path to Causal Gravity-Matter Systems**

- Description: Published.
- Type of collaboration: Joint research
- Researchers:
  - Prof. Reiko Toriumi, OIST
  - Prof. Antonio Pereira, Radboud University (the Netherlands) and Universidade Federal Fluminense (Brazil)
  - Juan Abranches, OIST

## **2.17 Supersymmetric Heisenberg model**

- Description: Ongoing.
- Type of collaboration: Joint research
- Researchers:
  - Dr. Nicolas Delporte, OIST
  - Dr. Jonas Sonnenschein, OIST

## **2.18 Dirac walks on regular trees**

- Description: Published.
- Type of collaboration: Joint research
- Researchers:
  - Mr. Saswato Sen, OIST
  - Dr. Nicolas Delporte, OIST
  - Prof. Reiko Toriumi, OIST

## **2.19 One-loop beta-functions of quartic enhanced tensor field theories**

- Description: Published.
- Type of collaboration: Joint research
- Researchers:

- Prof. Reiko Toriumi, OIST
- Prof. Joseph Ben Geloun, Laboratoire d'Informatique de Paris Nord, Université Paris 13, France

## 2.20 Keeler's Futurama Permutation Problem in type B and other variations

- Description: Ongoing.
- Type of collaboration: Joint research
- Researchers:
  - Dr. Christopher Chung, OIST and Xiamen University Malaysia
  - Juan Abranches, OIST

## 3. Activities and Findings

### 3.1 From Polymatroids to Delta-polymatroids

We introduce the notion of delta-polymatroid and the notion of hyper-quasitree, which set is proven to be convex. The concept of hyper-quasitree is motivated by the fact that the trees of a graph are the bases of a matroid and in the same way the bases of polymatroid are the hyper-trees of a hypergraph. We believe that the hyper-quasitrees will play the role of the bases of a delta-polymatroid which invariant will follow the fact that polymatroid invariant automatically gives a hypergraph invariant. The primary goal of this collaborative project is to answer the question, "How much of delta-matroid theory can be lifted to delta-polymatroid, and what kind?" This will allow us to investigate delta-polymatroid invariants in terms of graph invariants. This topic is mainly motivated by the fact that polymatroids are an abstraction of hypergraphs, just as matroids are an abstraction of graphs and delta-matroids are abstraction of ribbon graphs or graphs embedded in surfaces. Showing therefore a strong interconnection between graphs, hypergraphs, matroids, delta-matroids, polymatroids and surely delta-polymatroids

### 3.2 Classifying Graphs for Higher Grades for Multi-orientable Multi-Matrix Model

We studied in [Ann. Inst. Henri Poincaré D 9, 367-433, (2022)], a complex multi-matrix model with  $U(N)^2 \times O(D)$  symmetry, and whose double scaling limit where simultaneously the large- $N$  and large- $D$  limits were taken while keeping the ratio  $N/\sqrt{D} = M$  finite and fixed. In this double scaling limit, the complete recursive characterization of the Feynman graphs of arbitrary genus for the leading order grade  $\ell=0$  was achieved. In this current study, we classify the higher order graphs in  $\ell$ . More specifically,  $\ell=1$  and  $\ell=2$  with arbitrary genus, in addition to a specific

class of two-particle-irreducible (2PI) graphs for higher  $\ell \geq 3$  but with genus zero. Furthermore, we demonstrate that counting the 2PI graphs with a single  $O(D)$ -loop corresponds to enumerating the alternating knots using the Rolfsen's table results, performing a connected sum, or Tait flyping moves on them.

### 3.3 On Gross, Mansour and Tucker Conjecture for $\Delta$ -matroids

Gross, Mansour, and Tucker introduced the partial-duality polynomial of a ribbon graph [Distributions, European J. Combin. 86, 1--20, 2020], the generating function enumerating partial duals by the Euler genus. Chmutov and Vignes-Tourneret wondered if this polynomial and its conjectured properties would hold for general delta-matroids, which are combinatorial abstractions of ribbon graphs. Yan and Jin contributed to this inquiry by identifying a subset of delta-matroids-specifically, even normal binary ones-whose twist polynomials are characterized by a singular term. Building upon this foundation, the current paper expands the scope of the investigation to encompass even non-binary delta-matroids, revealing that none of them have width-changing twists.

### 3.4 N-cutoff Regularization for Fields on Hyperbolic Space

We apply a novel background independent regularization scheme, the N-cutoffs, to self-consistently quantize scalar and metric fluctuations on the maximally symmetric but non-compact hyperbolic space. For quantum matter fields on a classical background or full Quantum Einstein Gravity (regarded here as an effective field theory) treated in the background field formalism, the N-cutoff is an ultraviolet regularization of the fields' mode content that is independent of the background hyperbolic space metric. For each  $N > 0$ , the regularized system backreacts on the geometry to dynamically determine the self-consistent background metric. The limit in which the regularization is removed then automatically yields the 'physically correct' spacetime on which the resulting quantum field theory lives. When self-consistently quantized with the N-cutoff, we find that without any fine-tuning of parameters, the vacuum fluctuations of scalar and (linearized) graviton fields do not lead to the usual cosmological constant problem of a curvature singularity. Instead, the presence of increasingly many field modes tends to reduce the negative curvature of hyperbolic space, leading to vanishing values in the limit of removing the cutoff.

### 3.5 States of Low Energy on Bianchi I spacetimes

States of Low Energy are a class of exact Hadamard states for free quantum fields on cosmological spacetimes whose structure is fixed at *all* scales by a minimization principle. The original construction was for Friedmann-Lemaître geometries and is here generalized to anisotropic Bianchi I geometries relevant to primordial cosmology. In addition to proving the Hadamard property, systematic series expansions in the infrared and ultraviolet are developed. The infrared expansion is convergent and induces in the massless case a leading

spatial long distance decay that is always Minkowski-like but anisotropy modulated. The ultraviolet expansion is shown to be equivalent to the Hadamard property, and a non-recursive formula for its coefficients is presented.

### **3.6 A Wick Rotated Heat Kernel for Admissible Complex Metrics**

A Wick rotation in the lapse (not in time) is introduced for real manifolds admitting a co-dimension one foliation that interpolates between Riemannian and Lorentzian metrics. The definition refers to a fiducial foliation but covariance under foliation changing diffeomorphisms is ensured. In particular, the complex metrics are admissible in the sense of Kontsevich-Segal in all (fiducial and non-fiducial) foliations. This setting is used to construct a Wick rotated heat semigroup, which remains well-defined into the near Lorentzian regime. Among the results established for the Wick rotated version are: (i) existence as an analytic semigroup uniquely determined by its sectorial generator. (ii) construction of an integral kernel that is jointly smooth in the semigroup time and both spacetime arguments. (iii) existence of an asymptotic expansion for the kernel's diagonal in powers of the semigroup time whose coefficients are the Seeley-deWitt coefficients evaluated on the complex metrics. (iv) emergence of a Schrodinger evolution group in the strict Lorentzian limit. The toolbox includes local regularity results for admissible complex metrics.

### **3.7 Exact Quantization and Phase Transition**

In this research project, we explore a systematic extension of the resurgence properties to all phases in a given quantum theory. In our investigations, we focus on the exact quantization of one dimensional quantum mechanical systems using the Exact WKB method where the quantization conditions are linked to the geometry of Stokes diagrams. We observed that while different phases are associated to different geometries, they are, in fact, connected to each other by a smooth deformation of Stokes diagrams. With this observation, we showed that the form of the quantization condition of symmetric potentials stays the same. In addition to that we also showed while the role of WKB cycles changes at critical point (top of a barrier), their resurgence structure remains intact. We plan to publish our findings soon and continue our analysis with asymmetric wells as well as further investigations of phase transition phenomenon of various quantum system in the Exact WKB perspective.

### **3.8 Phase Transition from Perturbative Expansion**

In this project, I am investigating the (2nd order) phase transition phenomenon in the presence of spatially dependent electric fields with an aim to understand the universality near the critical point using solely the perturbative expansion and resurgent summation techniques.



### **3.9 Tensor Eigenvalues and Free Probability**

Based on a statistical physics reformulation of the resolvent of an operator that allows a more explicit control of concentration properties at large  $N$ , we are developing a free probabilistic theory of random tensors and its relation to the spectra of product of random matrices. The guiding intuition is that Fuss-Catalan numbers are present at the leading order moments in both cases. We also study second order fluctuations to contrast them with the well-known correlations of random matrices.

### **3.10 The Edge of Random Tensor Eigenvalues with Deviation**

Continuing previous work of the second author, we have in arXiv:2405.07731, compared the signed and genuine spectral distribution of random symmetric tensors of order 3 in the presence of a Gaussian deviation in the eigenvector equations. We have shown that the signed distribution, allowing an exact expression is correctly keeping track of the largest eigenvalue of the genuine one. It appeared that as the variance of the noise increased, an outlier emerged from the bulk of the spectrum, to later merge with the corresponding zero eigenvalue.

### **3.11 Power Iteration for Random Tensors**

The spiked tensor model has raised recently interest for presenting a theoretical and practical challenge in data analysis that involves recovering a signal hidden in a random tensor. There seems to be a gap between the information theoretic threshold and what current best polynomial time algorithms are able to retrieve. We are formulating this problem in a field theoretical way and hope to provide insight into the effectiveness of iterative algorithms where numerical tests and probabilistic bounds seem to clash.

### **3.12 Renormalization for Quantum Fields on Random Geometries**

We are developing a non-perturbative renormalization group of scalar and fermionic field theories suitable for a random geometric background that possesses a locally tree-like structure. In that regime, a good control over the heat-kernel spectrum is available, leading to flow equations where the dependence of the spectral dimension of the geometry is explicit. One can then incorporate (geometrical) loop corrections, as recently done for the Ising model on random graphs (arXiv:2403.01171).

### **3.13 Numerical Simulation of JT Gravity at Finite Cutoff**

JT gravity is a theory of two dimensional random surfaces of constant curvature. The limit of infinite cut-off has been well studied and is integrable due to a Schwarzian action. The case of finite cut-off is much more subtle, and

recent progress on the disk topology has been done in arXiv:2404.03748. Using a holographic rewriting of the bulk metric as a random scalar field living on the circle boundary, we approach the problem from numerical Monte Carlo simulations, that give access to the full partition function of the model. Besides, this random field forms a new universality class of random process, distinct from the Brownian motion and the self-avoiding walk, with new critical exponents that we investigate.

### **3.14 Harer-Zagier transform for families of twisted hyperbolic knots**

Knot matrix models are defined such that the averages of characters are equal to knot polynomials. However, an explicit measure for the average is only known for the special case of torus knots, while for general knots it remains an open problem. Our aim is to extend this definition to hyperbolic knots. In achieving this, a very useful tool that originates from random matrix theory, is the Harer-Zagier transform, which is a discrete version of a Laplace transform. Its factorisability seems to grant the superintegrability required for such an extension. Therefore, we applied this transform to the HOMFLY—PT polynomial for several families of twisted hyperbolic knots. Among them, we found a family of pretzel knots that also enjoys the factorisability property. Moreover, we studied the structure of their zero loci, which seems to have an interesting connection with ADE singularity theory.

### **3.15 Knot invariants from a matrix model perspective**

Motivated by our previous study, we launched on a journey of deeper exploration of the Harer-Zagier (HZ) transform of knot invariants and its factorisability properties. In the case of the HOMFLY—PT polynomial, we conjecture that such factorisability occurs if and only if, for a given knot, its HOMFLY—PT and Kauffman polynomials are related in a certain way. Moreover, we observed an interesting connection between the HZ transform and Khovanov homology. We also suggest that under a certain modification of the HZ function it can be related to Number Theory via modular forms.

### **3.16 Dually Weighted Multi-matrix Models as a Path to Causal Gravity-Matter Systems**

We introduce a dually-weighted multi-matrix model that for a suitable choice of weights reproduce two-dimensional Causal Dynamical Triangulations (CDT) coupled to the Ising model. When Ising degrees of freedom are removed, this model corresponds to the CDT-matrix model introduced by Benedetti and Henson [Phys. Lett. B 678, 222 (2009)]. We present exact as well as approximate results for the Gaussian averages of characters of a Hermitian matrix  $A$  and  $A^2$  for a given representation and establish the present limitations that prevent us to solve the model analytically. This sets the stage for the formulation of more sophisticated matter models coupled to two-dimensional CDT as dually weighted multi-matrix models providing a complementary view to the standard simplicial formulation of CDT-matter models.

### 3.17 Supersymmetric Heisenberg model

We are looking at field theoretic formulations of toy models for quantum magnetism. The problem is to understand the structure of the ground state and the fluctuations around it at a quantum critical point, separating an antiferromagnetic phase from a disordered phase. More precisely, we are looking at a variant of the Heisenberg model on a  $2d$  lattice, with a quartic interaction between spin variables, combining fermionic (suitable to describe the heavy Fermi liquid phase) and bosonic (suitable for the magnetised phase) operators, with a hard-core constraint on the bosons, restricting the particles to the physical states. In addition, a supersymmetry relating bosons to fermions simplifies the analysis using the superfield formalism as well as providing a way to connect continuously the seemingly antithetical phases of Fermi liquid and magnet. A large  $N$  limit of  $Sp(N)$  symmetry of the model was studied in [arXiv:1508.07861], that we would like to explore further, computing further correlations functions as well as Wilson loops of the gauged  $Sp(N)$  that would indicate a deconfined phase.

### 3.18 Dirac walks on regular trees

Analogous to expansion of two-point function in terms of random walks for a scalar field theory, we developed a similar tool for spin-less fermions on trees in terms of Dirac walks. Dirac walks are signed walks on vertex to edge, with a sign attached to each step owing to relative orientation of the edge and vertex involved. We computed explicit expressions for the free two-point, function, spectrum, local density of states and spectral dimension with respect to Dirac walks. Notably, we find that the spectrum of fermions develops a gap, and that the spectral dimension of the Dirac walks matches that of the simple random walk ( $d_s = 3$ ).

### 3.19 One-loop beta-functions of quartic enhanced tensor field theories

Enhanced tensor field theories (eTFT) have dominant graphs that differ from the melonic graphs of conventional tensor field theories. They therefore describe pertinent candidates to escape the so-called branched polymer phase, the universal geometry found for tensor models. For generic order  $d$  of the tensor field, we compute the perturbative beta-functions at one-loop of two just-renormalisable quartic eTFT coined by  $++$  or  $\times$ , depending on their vertex weights. The model  $++$  has two quartic coupling constants ( $\lambda$ ,  $\lambda_+$ ), and two 2-point couplings (mass,  $Z_a$ ). Meanwhile, the model  $\times$  has two quartic coupling constants ( $\lambda$ ,  $\lambda_{\times}$ ) and three 2-point couplings (mass,  $Z_a$ ). Meanwhile, the model  $\times$  has two quartic coupling constants ( $\lambda$ ,  $\lambda_+$ ) and three 2-point couplings (mass,  $Z_a$ ,  $Z_{2a}$ ). At all orders, both models have a constant wave function renormalisation:  $Z = 1$  and therefore no anomalous dimension. Despite such peculiar behavior, both models acquire nontrivial radiative corrections for the coupling constants. The RG flow of the model  $++$  exhibits a particular asymptotic safety:  $\lambda_+$  is marginal without corrections thus is a fixed point of arbitrary constant value. All remaining couplings determine relevant directions and get suppressed in the UV. Concerning the model  $\times$ ,

$\lambda_+$  is marginal and again a fixed point (arbitrary constant value),  $\lambda$ ,  $\mu$ , and  $Z_a$  are all relevant couplings and flow to 0. Meanwhile,  $Z_{2a}$  is a marginal coupling and becomes a linear function of the time scale. This model can neither be called asymptotically safe nor free.

## 3.20 Keeler's Futurama Permutation Problem in type B and other variations

In this collaborative project, we extend a theorem originally for the symmetric group, Keeler's theorem, to the signed symmetric group. The  $N$ th symmetric group  $S_N$  can be defined as the group of bijective functions from the set  $\{1, 2, \dots, N\}$  to itself, while the  $N$ th signed symmetric group  $SW_N$  is the group of odd bijective functions from the set  $\{-N, -N+1, \dots, N\}$  to itself. The original theorem gives a factorization of a permutations in  $S_N$  as products of distinct transpositions in  $S_{N+2} \setminus S_N$ . Our result extends this theorem to  $SW_N$ , where we factorize any element of  $SW_N$  into a product of transpositions, for an appropriate generalized notion of transpositions.

## 4. Publications

### 4.1 Journals

1. Banerjee, R., Niedermaier, M., States of Low Energy on Bianchi I spacetimes, *J. Math. Phys.*, doi: 10.1063/5.0160180 (2023).
2. Banerjee, R., Becker, M., Ferrero, R., N-cutoff regularization for fields on hyperbolic space, *Phys. Rev. D.*, doi: 10.1103/PhysRevD.109.025008 (2024).
3. R. C. Avohou, J. Ben Geloun and R. Toriumi, Counting  $U(N)^r O(N)^q$  invariants and tensor model observables, arXiv:2404.16404 [hep-th].
4. R. C. Avohou, On a conjecture of Gross, Mansour and Tucker for  $\Delta$ -matroids, arXiv:2404.13839 [math.CO].
5. R. C. Avohou, R. Toriumi and M. Vanraeynest, Classification of higher grade  $SIS$  graphs for  $U(N)^2 \times O(D)$  multi-matrix models, arXiv:2310.13789 [math-ph].
6. Delporte, N., Sen, S., Toriumi, R., Dirac walks on regular trees, *accepted in Journal of Physics A: Mathematical and Theoretical*, doi: 10.1088/1751-8121/ad4d2e [arXiv:2312.10881 [cond-mat]].
7. Delporte, N., Sasakura, N., The Edge of Random Tensor Eigenvalues with Deviation, [arXiv:2405.07731 [hep-th]].
8. Petrou, A. and Hikami, S. Harer–Zagier transform of the HOMFLY–PT polynomial for families of twisted hyperbolic knots, *J. Phys. A: Math. Theor.* 57: 205204. <https://doi.org/10.1088/1751-8121/ad421b> (2024)
9. Abranches, J.L.A., Pereira, A.D. & Toriumi, R. Dually Weighted Multi-matrix Models as a Path to Causal Gravity-Matter Systems. *Ann. Henri Poincaré* (2024).
10. Ben Geloun, J., Toriumi, R. One-loop beta-functions of quartic enhanced tensor field theories, *J. Phys. A: Math Theor.* 57 015401 (2023).

## 4.2 Books and other one-time publications

Nothing to report

## 4.3 Oral and Poster Presentations

([NOTE] \*Seminars and workshops by OIST faculty/unit members (either with or without other speakers), either at OIST or at other institutions than OIST, should be included in the 4.3 Oral and Poster Presentations.

1. Banerjee, R., *Self-consistent field quantization through sequences of gravity coupled approximants*, invited remote seminar, University of Nottingham, United Kingdom (2023).
2. Banerjee, R., *Wick rotating the heat kernel*, invited seminar, University of Leipzig, Germany (2023).
3. Banerjee, R., *Lorentzian functional renormalization group: Hadamard property and state (in)-dependence*, conference talk at Quantum Gravity 2023, Radboud University, The Netherlands (2023).
4. Banerjee, R., *Wick rotating the heat kernel*, invited seminar, Erwin Schrodinger Institute, Austria (2023).
5. Banerjee, R., *Wick rotating the heat kernel*, Geometric PDE and Applied Analysis Seminar, OIST, Japan (2023).
6. Banerjee, R., *From Riemannian to Lorentzian manifolds: complex metrics and the Wick rotated heat semigroup*, conference talk at Himeji Conference on Partial Differential Equations, Himeji, Japan (2024).
7. Banerjee, R. *Wick rotation in the lapse: admissible complex metrics and the Wick rotated heat kernel*, invited seminar, Institut Henri Poincare, Paris (2024).
8. Pazarbaşı, C. *A real time dependent approach to Schwinger mechanism*, conference talk, OIST, Okinawa (2023).
9. Pazarbaşı, C. *A time dependent approach to semi-classical approximation*, invited seminar, Kindai University, Osaka (2023).
10. Pazarbaşı, C. *A time dependent approach to semi-classical approximation*, invited seminar, Kindai University, Osaka (2023).
11. Pazarbaşı, C. *An introduction to Exact WKB method*, Boğaziçi University, İstanbul (2024)
12. Delporte, N., *Tensor models: an overview of non-perturbative results*. Presentation at "Invitation to Recursion, Resurgence and Combinatorics", OIST, Okinawa, From 4<sup>th</sup> to 14<sup>th</sup> of April (2023).
13. Delporte, N., *Eigenvalues of Random Tensors with Field Theoretic Methods*. Poster at "Random Matrices and Applications", RIMS, Kyoto, From 5<sup>th</sup> to 9<sup>th</sup> of June (2023).
14. Delporte, N., *Peeking at quantum gravity with self-overlapping curves*. Presentation at "Frontiers in Nonlinear Differential Equations and Stokes Phenomena", OIST, Okinawa, From 5<sup>th</sup> to 9<sup>th</sup> of September (2023).
15. Delporte, N., *JT gravity at finite cutoff with self overlapping curves*. Poster at "YIPQS long-term workshop: Quantum Information, Quantum Matter and Quantum Gravity", YITP, Kyoto, From 11<sup>th</sup> of September to 6<sup>th</sup> of October (2023).
16. Delporte, N., *Eigenvalues: from random matrices to tensors*. Presentation at "Non-Commutative Probability Theory, Random Matrix Theory and their Applications", ISM, Tokyo, From 8<sup>th</sup> to 11<sup>th</sup> of November (2023).

17. Delporte, N., Self-overlapping curves: geometry and statistics. Presentation at “6th Bangkok Workshop on Discrete Geometry, Dynamics and Statistics”, Chulalongkorn University, Bangkok, Thailand, From 8<sup>th</sup> to 12<sup>th</sup> of January (2024).
18. Petrou A. *Harer-Zagier formulas for families of twisted hyperbolic knots*, Presentation at “University of Tokyo and OIST Joint Symposium of Knot Theory”, University of Tokyo, 11 September (2023)
19. Petrou A., *Harer-Zagier formulas for families of twisted hyperbolic knots*, Silver Workshop VI, Okinawa, Japan, Country, 7-9 August (2023).
20. Petrou A., *The Harer-Zagier transform of the HOMFLY-PT polynomial*, Presentation at "The 19th east asian conference of geometric topology", Kyoto, Japan, 19-22 February (2024).
21. Petrou A., *The Harer-Zagier transform of the HOMFLY-PT polynomial*, Presentation at "Knot theory: LMO invariants and related topics", Okinawa, Japan, 9-11 March (2024)
22. Remi C. Avohou, *On partial dual polynomial of ribbon graphs*, Seminar, Cotonou, Benin, (2023).
23. Remi C. Avohou, *Brauer graph, and Brauer hypergraph algebras*, Seminar, Fukuoka, Japan, (2024).
24. Remi C. Avohou, *Matroids, and delta-matroids*, Seminar, L'IPN, Paris, France (2024).
25. S. Sen , Talk titled "QFT on trees" in Bangkok workshop on Discrete geometry, Dynamics and Statistics on Jan 8 2024, Chulalongkorn University, Bangkok, Thailand
26. S.Sen, Talk titled "QFT on trees" in Majorana-Raychaudhri Seminars on Jan 19 2024, online
27. Abranches, J. L. A, *Dually weighted multi-matrix models as a path to causal gravity-matter systems*, Presentation at “Virtual Tensor Journal Club”, 2024/02/21
28. Abranches, J. L. A, *Matrix Model for Causal Dynamical Triangulations with Ising Model: solution attempts and mathematical limitations*, Poster presentation at “Puzzles in the Quantum Gravity Landscape: viewpoints from different approaches”, Perimeter Institute, Canada, 2023/10/23 and 2023/10/24
29. Abranches, J. L. A, *Matrix Model for Causal Dynamical Triangulations with Ising Model: solution attempts and mathematical limitations*, Presentation at “Quantum Gravity in Bourdeaux 2023”, LaBRI, France, 2023/07/07
30. Toriumi, R, *Renormalisation of enhanced quartic tensor field theories*, talk at “Algebraic, analytic, geometric structures emerging from quantum field theory” conference, Chengdu, Sichuan University, China 11-15 March 2024
31. Toriumi, R, *Renormalisation of enhanced quartic tensor field theories*, talk at “From perturbative to non-perturbative QFT” conference, Muenster, University of Muenster, Germany, 14-16 June 2023
32. Toriumi, R, *Trisections in tensor models*, talk at “Random Geometry in Heidelberg” workshop, Heidelberg, University of Heidelberg, Germany, 16-20 May 2023

## 5. Intellectual Property Rights and Other Specific Achievements

Nothing to report

## 6. Meetings and Events

### 6.1 The Discrete Dirac operator and the mass of simple and higher-order networks

- Date: February 19, 2024
- Venue: Zoom
- Speakers:
  - Prof. Ginestra Bianconi (Queen Mary University London)

### 6.2 Graded locally semialgebraic spaces and graded Nash manifolds

- Date: August 18, 2023
- Venue: OIST Campus Lab4 F01
- Speakers:
  - Prof. Mahir Bilen Can (Tulane University)

### 6.3 Four-sided pegs fitting round holes fit all smooth holes

- Date: August 18, 2023
- Venue: OIST Campus Lab4 F01
- Speakers:
  - Prof. Andrew Lobb (Durham University)

### 6.4 New trends of conformal theory from probability to gravity

- Date: July 31 to August 4, 2023
- Venue: OIST Campus Lab4 E48
  - Organizers:
    - Prof. Shinobu Hikami (OIST)
  - Co-organizers:
    - Assist Prof. Reiko Toriumi (OIST)
    - Dr. Nicolas Delporte (OIST)
- Speakers:
  - Prof. Laurent Baulieu (LPTHE)
  - Assist Prof. Timothy Budd (Radboud Univ.)
  - Severin Charbonnier (Geneve Univ.) (online)
  - Prof. Bertrand Duplantier (IPhT, U Paris Saclay)
  - Nina Holden (Courant Inst.) (online)

- Motoko Kato (Ryukyu Univ.)
- Prof. Makoto Katori (Chuo Univ.)
- Dr. Shota Komatsu (CERN)
- Assist Prof. Wenliang Li (Sun Yat Sen Univ.) (online)
- Eveliina Peltola (Aalto Univ.)
- Assist Prof. Kazuhiro Saka (Meiji Gakuin)
- Xiaobing Sheng (OIST)
- Assist Prof. Hirohiko Shimada (Tsuyama Nat. Col. Tech.)
- Dr. Hidehiko Shimada (Yukawa Institute Kyoto)
- Assist Prof. Yiling Wang (IHES)
- Noriko Yui (Queen's Univ.)

## 6.5 Dimensional reduction in causal sets

- Date: July 24, 2023
- Venue: OIST Campus Lab4 E01
- Speakers:
  - Prof. David Meyer (UC San Diego)

## 6.6 Invitation to Recursion, Resurgence, and Combinatorics

- Date: April 7 to 10, 2023
- Venue: OIST Campus Lab4 E48
  - Co-organizers:
    - Dr. Nicolas Delporte (OIST)
    - Dr. Kento Osuga (University of Tokyo)
- Speakers:
  - Dr. Nezhla Aghaee (SDU)
  - Dr. Ioana Coman (Kavli IPMU)
  - Prof. Maciej Dolega (IMPAN)
  - Prof. Elba Garcia-Failde (Sorbonne University)
  - Dr. Alessandro Giacchetto (IPhT)
  - Dr. Paolo Gregori (IPhT Paris-Saclay)
  - Dr. Sabine Harribey (Nordita)
  - Prof. Yasuyuki Hatsuda (Rikkyo University)
  - Prof. Kohei Iwaki (University of Tokyo)
  - Prof. Taro Kimura (IMB/UBFC)
  - Dr. Reinier Kramer (University of Alberta)
  - Prof. Olivier Marchal (Université Jean Monnet)
  - Prof. Tatsuhiro Misumi (Kindai University)



- Dr. Yuto Moriwaki (Riken)
- Prof. Hajime Nagoya (Kanazawa university)
- Dr. Gergo Nemes (Department of Physics, Tokyo Metropolitan University)
- Dr. Romain Pascalie (ULB)
- Dr. Carlos Perez Sanchez (University of Heidelberg)
- Prof. Sergey Shadrin (Universiteit van Amsterdam)
- Prof. Raimar Wulkenhaar (Mathematical Institute, University of Münster)
- Dimitrios Mitsios (IPhT CEA, Saclay)
- Davide Lettera (Heidelberg University)
- Dr. Nikita Nikolaev (University of Birmingham)
- Dr. Omar Kidwai (University of Birmingham)

## 7. Other

Nothing to report.