FY2022 Annual Report

Gravity, Quantum Geometry and Field Theory Unit

Assistant Professor Reiko Toriumi

Abstract

Gravity, Quantum Geometry and Field Theory Unit studied the topics in quantum gravity. In particular, members are interested in random geometrical and field theoretical approaches both perturbative and nonperturbative methods. The research of the Unit intersects with a wide range of mathematics, including probability, combinatorics, topology, knot theory, reprentation theory, etc.

1. Staff

- Dr. Reiko Toriumi, Group Leader
- Dr. Nicolas Delporte, Postdoc
- Dr. Remi Cocou Avohou, Postdoc
- Dr. Rudrajit Banerjee, Postdoc
- Dr. Cihan Pazarbaşı, Postdoc
- Saswato Sen, Graduate Student
- Andreani Petrou, Graduate Student
- Juan Abranches, Graduate Student
- Matthias Vancraeynest, Research Intern
- Chikako Sugiyama, Administrative Assistant
- Yukiko Nakagawa, Administrative Assistant
- Chiyo Eto, Administrative Assistant

2. Collaborations

2.1 Scaling solutions for asymptotically safe gravity

- Description: Published.
- Type of collaboration: Joint research
- Researchers:
    - Prof. Christof Wetterich, Heidelberg Univerity, Germany
    - Dr. Masatoshi Yamada, Heidelberg University, Germany
    - Saswato Sen, OIST

2.2 Random Fields and Random Geometries

- Description: Ongoing.
- Type of collaboration: Joint research
- Researchers:
    - Prof. Reiko Toriumi, OIST

    - Dr. Nicolas Delporte, OIST
    - Saswato Sen, OIST

2.3 On ribbon configurations

- Description: Ongoing.
- Type of collaboration: Joint research
- Researchers:
    - Prof. Vyacheslav Futorny (USP, Brazil)
    - Prof. Kostiantyn Iusenko (USP, Brazil)
    - Dr. Remi Avohou, OIST

2.4 Classifying graphs for higher grades for multi-orientable multi-matrix model

- Description: Ongoing.
- Type of collaboration: Joint research
- Researchers:
    - Prof. Reiko Toriumi, OIST
    - Dr. Remi Avohou, OIST
    - Matthias Vancraeynest, OIST

2.5 Gross, Mansour and Tucker conjecture for Δ-matroids

- Description: Ongoing.
- Type of collaboration: Joint research
- Researchers:
    - Prof. Fabien Vignes Tourneret, CNRS Lyon (France)
    - Dr. Remi Avohou, OIST

2.6 Computerization of the Recursive Derivative Expansion of Schrodinger/Heat Kernels

- Description: Ongoing.
- Type of collaboration: Solo research
- Researchers:
    - Dr. Cihan Pazarbaşı, OIST

2.7 Non-perturbative Aspects of Worldline QED

- Description: Ongoing.
- Type of collaboration: Solo research
- Researchers:
    - Dr. Cihan Pazarbaşı, OIST

2.8 Non-perturbative Aspects of Random Matrix Models

- Description: Ongoing.
- Type of collaboration: Joınt research
- Researchers:
    - Prof. Reiko Toriumi, OIST
    - Dr. Nicolas Delporte, OIST
    - Dr. Cihan Pazarbaşı, OIST
    - Julian De Vuyst, OIST

2.9 On the Solution to the CDT Matrix Model's Partition Function

- Description: Ongoing.
- Type of collaboration: Joınt research
- Researchers:
    - Prof. Reiko Toriumi, OIST
    - Juan Abranches, OIST

2.10 JT gravity at finite cutoff

- Description: Ongoing.
- Type of collaboration: Joınt research
- Researchers:
    - Prof. Frank Ferrari, ULB
    - Dr. Romain Pascalie, ULB
    - Dr. Nicolas Delporte, OIST

2.11 Eigenvalues of random tensors

- Description: Ongoing.
- Type of collaboration: Joınt research
- Researchers:
    - Prof. Reiko Toriumi, OIST
    - Prof. Benoit Collins, Kyoto U
    - Prof. Naoki Sasakura, YITP
    - Dr. Luca Lionni, Heidelberg University
    - Dr. Nicolas Delporte, OIST

2.12 N-cutoff regularization for fields on hyperbolic space

- Description: Ongoing.
- Type of collaboration: Joınt research

- Researchers:
    - Dr. Rudrajit Banerjee, OIST
    - Dr. Maximilian Becker, Radboud University
    - Renata Ferrero, JGU Mainz

2.13 States of Low Energy on cosmological spacetimes

- Description: Ongoing.
- Type of collaboration: Joint research
- Researchers:
    - Prof. Max Niedermaier, University of Pittsburgh
    - Dr. Rudrajit Banerjee, OIST

2.14 Knot invariants and matrix models

- Description: Ongoing.
- Type of collaboration: Joint research
- Researchers:
    - Prof. Shinobu Hikami, OIST
    - Prof. Reiko Toriumi, OIST
    - Andreani Petrou, OIST

2.15 One-loop beta-functions of quartic enhanced tensor field theories

- Description: Published.
- Type of collaboration: Joint research
- Researchers:
    - Prof. Reiko Toriumi, OIST
    - Prof. Joseph Ben Geloun, University Paris 13

3. Activities and Findings

3.1 Scaling solutions for asymptotically safe gravity

The project was initiated by Saswato Sen as a master student in Heidelberg University with Dr. Yamada and continued after joining OIST as a research intern and then as a rotation student. Within a general truncation of the functional flow equations for quantum gravity with up to four derivatives of the metric we have demonstrated the existence of a scaling solution or critical trajectory from the asymptotically free ultraviolet fixed point to the infrared fixed point.

3.2 Random fields and Random geometries

We are developing an approach to understand quantum matter on random graphs. We use correspondence of random walk statistics and matter correlators to understand scattering amplitudes of field theories on graphs. We have identified a suitable random walk which corresponds to fermionic fields on graphs. We are studying the behaviour of such a random walker on various geometries.

3.3 On ribbon configurations

We review the Brauer configuration algebra. Some results about n-angulation algebra, which generalizes the triangulation algebra, are obtained. We also found a full formula for the Cartan matrix of the Brauer algebra, allowing us to characterize the Brauer configuration of affine and finite type.

3.4 Classifying Graphs for higher grades for multi-orientable multi-matrix model

The key objective of this work is to extend some recent findings from [Ann.Inst.H.Poincare D Comb.Phys.Interact., 2022, 9 (2), pp.367-433]. This paper describes how to build all of the grade $l=0$ melon-free Feynman graphs of genus g from the family of grade $l=0$ melon-free Feynman graphs of genus $g'<g$. This study finds a reasonable generalization for grade $l=1$ melon-free Feynman graphs of genus g, which we hope to generalize to arbitrary grade melon-free Feynman graphs.

3.5 Gross, Mansour and Tucker conjecture for Δ-matroids

The classical Euler-Poincaré duality is generalized by the partial duality of ribbon graphs with respect to a subset of their edges. This operation frequently alters the genus. J.L. Gross, T. Mansour, and T.W. Tucker recently proposed that for any ribbon graph other than plane trees and their partial duals, there is a subset of edges whose partial duality changes the genus. Qi Yan and Xian'an Jin discovered a family of counterexamples. S. Chmutov and F. Vignes-Tourneret show that these are the only counterexamples, and they wonder whether the partial-dual genus polynomial and related conjectures would make sense for delta-matroids. The primary purpose of this research is to provide an answer to such a query.

3.6 Computerization of the Recursive Derivative Expansion of Schrodinger/Heat Kernels

Heat Kernels or their Wick rotated counterpart Schrodinger kernels (time evolution propagators) are two of the central objects in quantum mechanics and worldline QFT problems. In this project, in order to compute high order perturbative expansions of these objects, we constructed an efficient algorithm based on a recursive derivative expansion, which is an organized version of the Schwinger DeWitt expansion. Currently, we are

working on its generalization for practical uses in application to various problems in quantum mechanics, abalian gauge theories and curved space time QFT.

3.7 Non-perturbative Aspects of Worldline QED

Based on the worldline formalism, we investigated the non-perturbative properties of the QED vacuum. Using high order perturbative expansions and their Borel-Pade summation, we obtained non-perturbative contributions to "beyond" the leading order for arbitrary background potentials. We also properly established a previously unnoticed relationship between the sources of non-perturbative information of the worldline QED and the ones obtained from (exact) WKB method. We plan to use this observation and extend our investigation to various projects on the non-perturbative and resurgence properties of various problems on quantum mechanics and worldline QFT.

3.8 Non-perturbative Aspects of Random Matrix Models

We reviewed the non-perturbative aspects of Hermitian matrix models at 1 cut and 2 cuts using their eigenvalue decomposition. We also worked on the relationship between the statistical physics of the eigenvalues and effective quantum mechanics which governs their dynamics. We are currently investigating the non-perturbative aspects of the effective quantum mechanics in relation with the observables of the random matrix models in various limits including the double-scaling limit where the observables are associated to 2D quantum gravity.

3.9 On the Solution to the CDT Matrix Model's Partition Function

We explore different solution methods for hermitian matrix models without unitary symmetry, with the goal of applying it to the unsolved problem of the partition function of the CDT matrix model. We work with representation theory properties such as character expansion and Schur-Weyl duality. We are able to find a solution in the large N limit, with the possible extension to finite N. This project started as an internship project that was continued as a rotation project, and is currently on writing process.

3.10 JT gravity at finite cutoff

We use analytical and numerical tools to characterise the phase space of two-dimensional geometries with boundary and fixed curvature, in terms of properties of their boundaries (corresponding to self-overlapping curves). We aim at making precise the regimes where the Schwarzian approximation (for hyperbolic metrics) and self-avoiding walk hold and what lies beyond. These curves are still relatively new objects and good sets of parameters are being searched for.

3.11 Eigenvalues of random tensors

We try bridge different approaches to eigenvalues of random tensors and using field theoretic techniques (sypersymmetric formalism and large-N approximations) to derive the eigenvalue distribution at leading order,

the law of the largest eigenvalue, their correlations. Subleading orders are also important to see how far any relation to free probability can go.

3.12 N-cutoff regularization for fields on hyperbolic space

We investigate a novel self-consistent quantization scheme, the N-cutoffs, for scalar and metric fluctuations on the maximally symmetric but non-compact hyperbolic space. The N-cutoffs are a regularization on the spectrum of the fields' fluctuation modes, and we find that the inclusion of increasingly many modes tends to drive the the negative curvature of hyperbolic space towards zero, leading to vanishing values when the cutoff is removed.

3.13 States of Low Energy on cosmological spacetimes

States of Low Energy are exact Hadamard states for free quantum fields on Friedmann-Robertson-Walker spacetimes. We extend this construction to a wide class of spacetimes relevant for primordial cosmology, where in addition to proving the Hadamard property, systematic series expansions in the infrared and ultraviolet regimes are developed.

3.14 Knot matrix models and skein theory

Knot matrix models are defined by equating knot polynomial invaianrts to averages of characters of representations. However, an explicit measure for the average is only known for the special case of torus knots and its generalization to other families of knots remains an open problem. Our aim is to extend this definition to hyperbolic knots, by exploiting mirror dymmetry between unitary and Hermitian ensembles. In parallel, we are trying to construct a recursive formula for the HOMFLYPT polynomial of torus knots via skein theory, which may be given an interpretation as creation/annhilation of fermions and bosons.

3.15 One-loop beta-functions of quartic enhanced tensor field theories

Enhanced tensor field theories (eTFT) have dominant graphs that do not correspond to melonic diagrams of ordinary tensor field theories. They therefore describe pertinent candidates to escape the so-called branched polymer phase, the universal geometry found for tensor models. For generic rank d of the tensor field, we compute the perturbative beta-functions at one-loop of two just-renormalizable quartic eTFT coined by + or x, depending on their vertex weights. The models + has two quartic coupling constants $(¥lambda, ¥lambda_+)$, and two 2-point couplings (mass, Z_a). Meanwhile, the model x has two quartic coupling constants $(¥lambda, ¥lambda_x)$ and three 2-point couplings (mass, Z_a, Z_{2a}). At all orders, both models have a constant wave function renormalization: $Z=1$ and therefore no anomalous dimension. Despite such peculiar behavior, both models acquire nontrivial radiative corrections for the coupling constants. The RG flow of the model $+$ exhibits neither asymptotic freedom nor the ordinary Landau ghost of phi^4_4 model: ¥lambda_+ is a fixed point and ¥lambda has linear behavior in time scale $t = ¥log(k/k_0)$. In the UV, the mass behaves likewise namely is linear in t, whereas Z_a decreases exponentially towards a constant value. For the model

x, both $¥lambda$ and $¥lambda_x$ do not flow, all remaining 2-point coupling constants are linear functions of the time scale in the UV.

4. Publications

4.1 Journals

1. Sen, S.;Wetterich, C.; Yamada, M. Scaling solutions for asymptotically safe gravity *JHEP* 02 (2023) 054 . SCALING SOLUTIONS FOR ASYMPTOTICALLY SAFE GRAVITY, DOI: 10.1007/JHEP02(2023)054
2. Ben Geloun, J.;Toriumi, R. *One-loop beta-functions of quartic enhanced tensor field theories* arXiv:2303.09829 DOI:10.48550/arXiv.2303.09829
3. Martini, R.;Toriumi, R. *Trisections in colored tensor models,* accepted to Annales de l'Institut Henri Poincare D: Combinatorics, Physics and their interactions DOI:10.4171/AIHPD/167

4.2 Books and other one-time publications

Nothing to report

4.3 Oral and Poster Presentations

1. Remi C. Avohou, *From Brauer graph algebra to Brauer configuration algebra*, workshop "Women at the intersection of mathematics and theoretical physics meet in Okinawa", Okinawa, Japan, 20-24 March 2023.
2. Andreani Petrou, *Towards knot matrix models for families of twisted hyperbolic knots*, Silver Workshop V, Okinawa, Japan, Country, 9-11 November 2022.
3. Andreani Petrou, *Towards knot matrix models for families of twisted hyperbolic knots*, workshop "Women at the intersection of mathematics and theoretical physics meet in Okinawa", Okinawa, Japan, 20-24 March 2023.
4. Nicolas Delporte, *A random walk approach to two dimensional quantum gravity*, invited seminar, LIPN, Paris, France, 14 February 2023.
5. Nicolas Delporte, *On aspects of two-dimensional quantum gravity*, invited seminar, LIPN, Paris, France, 14 February 2023.
6. Nicolas Delporte, *Peeking at quantum gravity with self-overlapping curves*, invited seminar, IHP, Paris, France, 15 February 2023.
7. Reiko Toriumi, *Trisections in colored tensor models*, workshop "Random Geometry in Heidelberg", Heidelberg University, Germany 16-20 May 2022.
8. Reiko Toriumi, *Trisections in colored tensor models*, workshop "Quantum Gravity and Random Geometry", Institute Henri Poincare, Paris, France 16-20 January 2023.
9. Rudrajit Banerjee, *The spatial Functional Renormalization Group and Hadamard states on cosmological spacetimes,* conference "Pheno 2022", University of Pittsburgh, USA, 9-11 May 2022.
10. Rudrajit Banerjee, *Wick rotating the heat kernel*, invited seminar, IMAPP Radboud University, Nijmegen, The Netherlands, 24th February 2023.

11. Rudrajit Banerjee, *Wick rotating the heat kernel*, invited seminar, JGU Mainz, Germany, 28th February 2023.

5. Intellectual Property Rights and Other Specific Achievements

Nothing to report

6. Meetings and Events

6.1 Vortex counting, WKB and the higher order Stokes phenomenon

- Date: March 28, 2023
- Venue: OIST Campus Lab4 E01
- Speaker: Dr. Samuel Crew (University of Bath)

6.2 An introduction to combinatorial compact quantum groups and free Bessel distributions

- Date: March 27, 2023
- Venue: OIST Campus Lab4 F01
- Speaker: Prof. Benoit Collins (Kyoto University)

6.3 Women at the Intersection of Mathematics and Theoretical Physics Meet in Okinawa

- Date: March 20 to 24, 2023
- Venue: OIST Campus Lab4 E48
- Co-organizers:
    - Dr. Shihoko Ishii (University of Tokyo)
    - Dr. Sylvie Paycha (University of Potsdam)
    - Dr. Susanne Reffert (University of Bern)
    - Dr. Kasia Rejzner (University of York)
    - Dr. Xiaodan Zhou (OIST)
- Speakers:
    - Dr. Yuanyuan Bao (University of Tokyo, Japan)
    - Dr. Xenia de la Ossa (Oxford University, UK)
    - Dr. Yukari Ito (IPMU, Japan)
    - Dr. Motoko Kato (Ryukyu University, Japan)
    - Dr. Keiko Kawamuro (University of Iowa, USA)
    - Dr. Yukiko Konishi (Tsuda Univesity, Japan)
    - Dr. Yolanda Lozano (Univesity of Oviedo, Spain)
    - Dr. Chihiro Matsui (University of Tokyo, Japan)
    - Dr. Silvia Penati (INFN Milano-Biccoca, Italy)
    - Dr. Makiko Sasada (University of Tokyo, Japan)
    - Dr. Simona Settepanella (University of Torino, Italy)

- Dr. Mayuko Yamashita (RIMS, Kyoto, Japan)

6.4 Refined BPS invariants via topological recursion

- Date: November 25, 2022
- Venue: OIST Campus Lab4 F01
- Speaker: Dr. Kento Osuga (University of Sheffield, UK)

6.5 Invitation to topological recursion and their applications

- Date: November 17, 2022
- Venue: OIST Campus Lab4 F01
- Speaker: Dr. Kento Osuga (University of Sheffield, UK)

6.6 Landau-Ginzburg analysis of (T)GFT models

- Date: September 15, 2022
- Venue: OIST Campus Lab4 E01
- Speaker: Dr. Luca Marchetti (Ludwig-Maximilians-Universität München)

6.7 Parametric resurgent trans-series and large N vortex counting

- Date: August 22, 2022
- Venue: OIST Campus Lab4 F01
- Speaker: Dr. Samuel Crew (University of Bath)

6.8 Combinatorics and knot invariants

- Date: June 24, 2022
- Venue: OIST Campus Lab4 F01
- Speaker: Dr. Robert Osburn (University College Dublin, Ireland)

7. Other

Women in Mathematics Photography Exhibits

- Date: March 20 to 24, 2023
- Venue: OIST Central Building LevelC, Tunnel Gallery
- Photographer: Noel Tovia Matoff