

FY2020 Annual Report

Gravity, Quantum Geometry and Field Theory Unit

Assistant Professor Reiko Toriumi

Abstract

Gravity, Quantum Geometry and Field Theory Unit studied the topics in quantum gravity. In particular, members are interested in random geometrical and field theoretical approaches, focusing on questions relating to topological and geometrical information.

1. Staff

- Dr. Reiko Toriumi, Assistant Professor
- Dr. Guilherme Sadovski, Postdoctoral Scholar
- Dr. Riccardo Martini, Postdoctoral Scholar
- Dr. Nicolas Delporte, Postdoctoral Scholar
- Ms. Ayumi Shimojima, Research Unit Administrator
- Ms. Yi-Shan Cheng, OIST Student (Sep.2020-Dec.2020)
- Mr. Samuel Cyrus Cure, OIST Student (Jan.2021-Apr.2021)
- Mr. Nico Fischer, OIST Student (Jan.2021-Apr.2021)

2. Collaborations

2.1 Eigenvalues of tensors in tensor models

- Description: Ongoing.
- Type of collaboration: Joint research
- Researchers:
 - Prof. Reiko Toriumi, OIST
 - Dr. Nicolas Delporte, OIST
 - Joren Brunekreef, Radboud University Nijmegen, the Netherlands
 - Samuel Cure, OIST

2.2 Topological recursion in the triple scaling limit of $U(N)^2 \times O(D)$ multi-matrix models

- Description: Ongoing.
- Type of collaboration: Joint research
- Researchers:
 - Prof. Reiko Toriumi, OIST
 - Dr. Nicolas Delporte, OIST
 - Dr. Kento Osuga, Sheffield University, UK

2.3 Universality classes in quantum gravity

- Description: Ongoing.
- Type of collaboration: Joint research
- Researchers:
 - Dr. Omar Zanusso, University of Pisa, Italy
 - Dr. Alessandro Ugoletti, Friedrich-Schiller University of Jena, Germany
 - Dr. Riccardo Martini, OIST
 - Francesco Del Porro, SISSA, Italy

2.4 Flat JT gravity

- Description: Ongoing.
- Type of collaboration: Joint research
- Researchers:
 - Dr. Nicolas Delporte, OIST
 - Prof. Frank Ferrari, Universite Libre de Bruxelles, Belgium B]
 - Dr. Romain Pescalie, Universite Libre de Bruxelles, Belgium

2.5 (In-)equivalence between holonomic and non-holonomic gravities

- Description: Ongoing.
- Type of collaboration: Joint research
- Researchers:
 - Dr. Guilherme Sadovski, OIST
 - Prof. Jorge Zanelli, Centro de Estudios Cientificos (CECs), Valdivia, Chile
 - Prof. Rodrigo F. Sobreiro, Universidade Federal Fluminense, Brazil

2.6 Renormalizable TQFT phase for gravity

- Description: Ongoing.
- Type of collaboration: Joint research
- Researchers:
 - Dr. Guilherme Sadovski, OIST
 - Dr. Octavio Junqueira, Universidade Federal do Rio de Janeiro, Brazil
 - Prof. Rodrigo F. Sobreiro, Universidade Federal Fluminense, Brazil

2.7 Trisections in tensor models

- Description: Ongoing.
- Type of collaboration: Joint research
- Researchers:
 - Prof. Reiko Toriumi, OIST

3. Activities and Findings

3.1 Eigenvalues of tensors in tensor models

The project is initiated by the OIST rotation student, Samuel Cure. The study investigates the eigenvalues of large symmetric random tensors and their statistical spectral properties. Main motivations lie in the context of tensor models approach to quantum gravity and geometry. Tensor models are natural generalisation, therefore higher dimensional analogue of matrix models, which in their continuum limit, has been shown to give Liouville quantum gravity (2-dimensions) with conformal matter. Many well developed techniques of matrix models are based on eigenvalues of matrices, which are not easily accessible in tensor models, since there is no good canonical notion of eigenvalues for tensors. Nevertheless, many definitions of eigenvalues for tensors exist and in principle, eigenvalues should capture some tensor invariants. It will be therefore useful to develop the understanding of eigenvalues of tensors in order to open up the wealth of tools available and well established in matrix models to apply to the tensor models. We based our investigation on the paper by Gurau "On the generalization of the Wigner semicircle law to real symmetric tensors" arXiv:2004.02660, which generalised the random matrices' Wigner semicircle law of the spectral density to real symmetric tensors using resolvents of p -spin glass models. We found the following aspects:

- Confirmed the melonic universality by changing the distribution to non-Gaussian measures (but still melonic) to verify that the Gurau's generalised spectral density remains unchanged.

- After numerical analyses, although in principle the relation between the spectral density and eigenvalues exists however not clear since there is no analogue of spectral theorem for tensors, we observed that the scalings of real eigenvalues, their support, and their distribution seem to differ in a pronounced manner.

Reflecting the above two findings, we are investigating further whether there exists the melonic universality in the distribution of eigenvalues.

3.2 Topological recursion in the triple scaling limit of $U(N)^2 \times O(D)$ multi-matrix models

Recent years have witnessed impressive progress regarding understanding correlation functions of matrix models of group size N through topological recursion. This recursion, derived from Loop or Schwinger-Dyson equations, allows to extract correlation functions of n -external points on embedded surface of genus g , from those of lower values of g and n . In the later paper [arXiv:2003.02100], studying multiscaling limits of a $O(D) \times U(N)^2$ tensor models, a new solvable regime has been obtained, dual to tree-like graphs. By analysing the n -point functions of this model in this particular regime, we are attempting to find an appropriate generalisation of the topological recursion that would also incorporate tensor models.

3.3 Gravity in $d = 2 + \epsilon$ dimensions and realizations of the diffeomorphisms group

Different realizations of the diffeomorphism group were investigated for theories of metric fields in the vicinity of $d=2$ dimensions. The two realizations of the symmetry group allow for different parametrization of the geometric degrees of freedom. The actions for the corresponding theories define equivalent classical theories which differ at the quantum level. The renormalization properties in the limit where ϵ vanishes can be detected via the form of the conformal anomaly in two dimensions and depend on a choice of renormalization scheme, nevertheless, the full set of available schemes clearly changes between the two formulations of quantum gravity. The analytic continuation of such theories for dimensions higher than $d=2$ was also studied and hints of a conformal window that includes the physical case of $d=4$ were highlighted.

3.4 Flat JT gravity

Given the interest that has sparked recently from the SYK model for two-dimensional hyperbolic geometries, made dynamical through the inclusion of a dilaton field, and their relevance for understanding the many faces of black holes, it seems a natural idea to study their flat counterpart. Lessons derived might as well provide a different point of view on the current factorization issue and the associated wormholes.

3.5 (In-)equivalence between holonomic and non-holonomic gravities

In collaboration with J. Zanelli (CECs, Chile) and R.F. Sobreiro (UFF, Brazil), Dr. Guilherme Sadovski investigated the possible scenarios in which a holonomic vs a non-holonomic frame description of gravity theories fail to be equivalent. It turns out that classically, the equivalence holds in a way that is independent of the particular dynamics or spacetime dimension. This includes more general setup of the Einstein-Cartan or Metric-Affine type. A geometric bundle-theoretical reasoning was given, uncovering the geometrical equivalence principle as culprit. Quantum mechanically, the equivalence holds as long as the equivalence principle holds. This is not something to be expected in the full non-perturbative regime of quantum gravity, where non-invertible configuration of the vielbein must be accounted for. In such scenario, the gauge-theoretical description of gravity unfolds from spacetime and one has to decide if gravity is spacetime geometry or a gauge theory. Pre-print is being finalized.

3.6 A renormalizable TQFT phase for quantum gravity

In collaboration with R.F. Sobreiro (UFF, Brazil) and O.C. Junqueira (UFRJ, Brazil), Dr. Guilherme Sadovski investigated the quantum stability of a cohomological TQFT that can be interpreted as symmetry-restored phase of gravity. The rich set of Ward identities guarantees that the theory is quantum mechanically stable and renormalizable. The connection to gravity is achieved via an explicit break of the topological BRST symmetry. Such break can possibly have origins in the coupling to topological matter or via a Higgs-like mechanism. Pre-print is being finalized.

3.7 Computing trisections from tensor models

Trisections are a powerful way to describe the topology of four dimensional manifolds. They were recently discovered to be the proper higher dimensional equivalent of Heegaard splittings. In fact, similarly to Heegaard diagrams, trisection diagrams allow to represent the topology of a four dimensional manifold in terms of lower dimensional objects, namely, a closed surface and a set of three closed curves on it. Studying trisections could lead to important development towards a better understanding of the gravitational path integral in four dimensions. We found a way to compute trisections in the PL-category from the purely combinatorial information encoded in tensor models for a generic graph. Our work provides an important extension of formalisms previously developed in the mathematical community. A draft including the complete results is currently under construction by Dr. Riccardo Martini and Prof. Reiko Toriumi.

4. Publications

4.1 Journals

1. Klitgaard, N.; Loll, R.; Reitz, M., Toriumi, R. "Geometric flux formula for the gravitational Wilson loop", [Classical and Quantum Gravity](#).
2. Benedetti, D.; Delporte, N. "Remarks on a melonic field theory with cubic interactions", [arXiv:2012.12238](#).
3. Benedetti, D.; Carrozza, S.; Toriumi, R.; Valette, G. "Multiple scaling limits of $U(N)^2 \times O(D)$ multi-matrix models", accepted and to appear in Annales de l'Institut Henri Poincaré D.

4.2 Books and other one-time publications

1. Preprint: Martini, R.; Del Porro, F.; Ugolotti, A.; Zanusso, O. "Gravity in $d = 2 + \epsilon$ dimensions and realizations of the diffeomorphisms group", [arXiv:2103.12421](#)

4.3 Oral and Poster Presentations

1. Sadovski, G. "Exotic smoothness and Physics" seminar in Google Meet, organized by Grupo de Fisica Teorica, Universidade Federal Fluminense, Brazil, May 25, 2020.
2. Sadovski, G. "Porque o espaco-tempo e' quadri-dimensional?" ("Why spacetime is four-dimensional?" in translation), [seminar in Google Meet](#), organized by Grupo de Informacao Quantica e Fisica Estatistica, Universidade Federal do Oeste da Bahia, Brazil, July 17, 2020.
3. Sadovski, G. "Why is the world 4-dimensional?", invited seminar, Theoretical Physics Seminar, OIST, Japan, Oct 9, 2020.
4. Toriumi, R. "Quantum Gravity, Quantum Observables, Wilson Loops, and Geometric Flux", OIST Faculty Lunch Seminar, Nov. 10, 2020
5. Delporte, N. "Tensor Field Theories: Renormalization and Random Geometry" by Zoom, OIST, Japan, Nov. 30, 2020.

6. Delporte, N. "Remarks on a melonic field theory with cubic interactions", [virtual Tensor Journal Club](#), February 10, 2021.

Colloquiums:

1. Sadovski, G. Simplicial homology, part IV. OIST, Japan, Apr. 10, 2020.
2. Cheng, Y. Colored Tensor Models Review. OIST, Japan, Oct. 5, 2020.
3. Cheng, Y. Colored Tensor Models Review. OIST, Japan, Oct. 12, 2020.
4. Cheng, Y. Colored Tensor Models Review. OIST, Japan, Oct. 19, 2020.
5. Martini, R. Boundary graphs in tensor models (Journal club). OIST, Japan, Oct. 23, 2020.
6. Cheng, Y. Colored Tensor Models Review. OIST, Japan, Oct. 29, 2020.
7. Cheng, Y. Colored Tensor Models Review. OIST, Japan, Nov. 6, 2020.
8. Cheng, Y. Colored Tensor Models Review. OIST, Japan, Nov. 9, 2020.
9. Sadovski, G. "Henry, L.L. Free electromagnetic fields on a compact Lie group manifold J. Math. Phys. (1977)" Nov 12, 2020.
10. Cheng, Y. Colored Tensor Models Review. OIST, Japan, Nov. 16, 2020.
11. Cheng, Y. Colored Tensor Models Review. OIST, Japan, Nov. 26, 2020.
12. Sadovski, G. "Streaming 1/2 of "Witten, E. Quantum gravity" Dec 15, 2020.
13. Sadovski, G. "Streaming 2/2 of "Witten, E. Quantum gravity" Dec 21, 2020.
14. Sadovski, G. "Bergshoeff, E. et al Carrol versus Galilei gravity" Jan 15, 2021.
15. Delporte, N. Tensor Overview I. OIST, Japan, Jan. 27, 2021.
16. Curé, S. Eigenvalues of Large Random Tensors I. OIST, Japan, Jan. 29, 2021.
17. Delporte, N. Tensor Overview II. OIST, Japan, Feb. 2, 2021.
18. Curé, S. Eigenvalues of Large Random Tensors II. OIST, Japan, Feb. 3, 2021.
19. Fisher, N. SYK Seminar, Part I: Introduction. OIST, Japan, Feb. 4, 2021.
20. Toriumi, R. Triple scaling of a multimatrix model $U(N)^2 \times O(D)$. OIST, Japan, Feb 5, 2021.
21. Toriumi, R. Triple scaling of a multimatrix model $U(N)^2 \times O(D)$, continued. OIST, Japan, Feb 12, 2021.
22. Fisher, N. SYK Seminar, Part II: Melons. OIST, Japan, Feb. 12, 2021.
23. Curé, S. Eigenvalues of Large Random Tensors III. OIST, Japan, Feb. 17, 2021.
24. Fisher, N. SYK Seminar, Part III: Effective Action. OIST, Japan, Feb. 18, 2021.
25. Curé, S. Eigenvalues of Large Random Tensors IV. OIST, Japan, Feb. 26, 2021.
26. Fisher, N. SYK Seminar, Part IV: Emergent IR Symmetry. OIST, Japan, Mar. 3, 2021.
27. Fisher, N. SYK Seminar, Part V: JT Gravity 1/2. OIST, Japan, Mar. 17, 2021.
28. Curé, S. Eigenvalues of Large Random Tensors IV. OIST, Japan, Mar. 25, 2021.
29. Fisher, N. SYK Seminar, Part VI: JT Gravity 2/2. OIST, Japan, Mar. 30, 2021.

5. Intellectual Property Rights and Other Specific Achievements

Nothing to report

6. Meetings and Events

6.1 Minicourse: "Morse Theory"

- Date: April 1, 2020
- Venue: OIST campus
- Speaker: Prof. Andrew Lobb (Durham University, and OIST)

6.2 Presidential Lecture "The Square Peg Problem"

- Date: July 30, 2020
- Venue: L4E48, OIST campus
- Speaker: Prof. Andrew Lobb (Durham University, and OIST)

6.3 Seminar: "An invitation to topological string theory"

- Date: Jan. 21, 2021
- Venue: L4F01, OIST Campus
- Speaker: Dr. Kento Osuga (University of Sheffield, UK)

6.4 Seminar: "Matrix models and topological recursion"

- Date: Jan. 21, 2021
- Venue: L4F01, OIST Campus
- Speaker: Dr. Kento Osuga (University of Sheffield, UK)

6.5 Seminar: "Matrix models and topological recursion II"

- Date: Feb. 9, 2021
- Venue: L4F01, OIST Campus
- Speaker: Dr. Kento Osuga (University of Sheffield, UK)

6.6 Seminar: "Kinematic algebra"

- Date: Feb. 9, 2021
- Venue: zoom
- Speaker: Dr. Tianheng Wang (Humboldt University Berlin, Germany)

6.7 Seminar: "Unitary matrix models"

- Date: Feb. 16, 2021
- Venue: zoom
- Speaker: Neetu (IISER, Bhopal, India)

6.8 Seminar: "Topological recursion for tensor models"

- Date: Feb. 20, 2021
- Venue: zoom
- Speaker: Prof. Valentin Bonzom (LIPN – Combinatorics, Algorithms and Interactions – Paris University 13)

6.9 Seminar: "Resurgent Properties of Minimal Strings and Painlevé Equations"

- Date: Feb. 23, 2021
- Venue: zoom
- Speaker: Robert Vega (Instituto Superior Tecnico, U Lisboa, Portugal)