# Self-Organization and Compositionality in Cognitive Brains: A Neuro-Robotics Study

## Jun Tani, Dept. of Electrical Engineering, KAIST

(*Proceedings of the IEEE*, Special Issue on Cognitive Dynamic Systems, Vol. 102, no. 4, pp. 586–605, 2014.)

## Abstract

The most pressing question about cognitive brains is how they support the compositionality that enables combinatorial manipulations of images, thoughts and actions. When addressing this problem with synthetic modeling, the conventional idea prevalent in artificial intelligence and cognitive science, generally, is to assume hybrid systems and corresponding neural network models, where higher-order cognition is realized by means of symbolic representation and lower sensory-motor processes by analogue processing. However, the crucial problem with such approaches is that the symbols represented at higher order cognitive levels cannot be grounded naturally in sensory-motor reality. The former are defined in a discrete space without any metric and the latter are defined in a continuous space with a physical metric. These, therefore, cannot directly interact with each other, regardless of the interface that is assigned between them. The proposal in the current paper is to reconstruct higher-order cognition by means of continuous neuro-dynamic systems that can elaborate delicate interactions with the sensory-motor level while sharing the same metric space. Our neuro-robotics experiments - including language-action associations and the learning of goal-directed actions - show that the compositionality necessary for higher-order cognitive tasks can be acquired by means of self-organizing dynamic structures, via interactive learning between the top-down intentional process of acting on the physical world and the bottom-up recognition of perceptual reality. Using robotic simulations, the current paper demonstrates that nonlinear dynamic phenomena, such as bifurcations and the chaotic dynamics induced by unstable fixed points, could play essential roles in realizing higher-order functions.

# Key words and index terms

self-organization, compositionality, neuro-robotics, chaos, cognition, dynamical systems, neural network models, sensory-motor systems

# **I. Introduction**

One of the amazing aspects of human brains is that they can generate diverse thoughts, images and actions through novel combinations of acquired knowledge and skills. Such cognitive competency is expressed well by the principle of compositionality, i.e. the meaning of the whole is a function of the meaning of the parts. As described by Gareth Evans [1] in regard to language, the principle of compositionality asserts that the meaning of a complex expression is determined by the meanings of its constituent expressions and the rules used to combine them, to wit, sentences are composed from sequences of words combined according to grammatical rules, and can be decomposed similarly. This central notion, that the whole can be decomposed into reusable parts (or primitives) by following rules, is applicable to other faculties, as well, such as to the generation of complex action routines. For example, the motor schemata theory [2] proposes that complex goal-directed actions can be decomposed into sequences of behavior primitives. Here, behavior primitives are commonly used behavior segments or motor programs.

From the preceding considerations, a crucial question emerges. How is compositionality realized in cognitive brains? Cognitive scientists have considered that such compositionality is a product of manipulations of arbitrary shapes of tokens within a symbolic system [3]. Because the manipulation of symbols which are, in themselves, without any physical dimensions such as weight, length, speed, or force is free from any constraints due to these physical dimensions on possible combinations of such symbols, a symbol system provided with recursive functionality achieves compositionality with an infinite range of possible expressions. However, studies on intelligent (or cognitive) robots have revealed that this framework, employing symbolic representation and manipulation, encounters problems when symbols are required to be grounded within the context of continuous sensory-motor flow [4]. This problem, the famous symbol grounding problem [3], [5], becomes crucial for cognitive robots especially when inconsistencies appear between what the symbol level represents in the top-down pathway and the reality which arises from the sensory-motor level in the bottom-up pathway. It is assumed that both levels should participate in attempts to resolve such conflicts via cooperative processes. This cooperation entails iterative interactions between the two levels through which optimal matching between them is sought dynamically. If one level pushes forward a little, the other should pull back elastically so that a point of compromise can be found through iterative dynamic interactions. Yet, this problem is not so easily solved through conventional hybrid approaches. The symbol systems defined in a discrete space are too rigid to afford such delicate interactions with the sensory-motor system. Moreover, this problem cannot be resolved

by simply implementing arbitrary interfaces between the two systems, because they simply do not share the same metric space for the interactions.

Confronted with these difficulties, it may be fruitful to inquire into how this problem is understood, and possibly solved, in terms of human brains. Specifically, we would like to know how neuronal structures enabling compositionality in the higher-order cognition level develop in human brains. With such information, we might better understand how such structures can remain adequately grounded in ongoing perceptual input. However, it is fair to say that these mechanisms are not yet exactly understood. Significant evidence has accumulated, nevertheless, for a convergent understanding that the prefrontal cortex is involved in compositionality due to its executive control of other parts of the brain [6]-[8]. Joachim Fuster writes in his textbook [8] that executive function is the ability to organize a sequence of actions toward a goal, an operation requiring compositionality as described previously. Another aspect of compositionality within human brains for which there is some evidence is that they utilize hierarchy in complex information processing. For instance, the visual recognition of complex objects is performed hierarchically, beginning with V1 and V2 for simple feature detection, and proceeding to the inferotemporal cortex for the integration of those features into more complex compositions [9], [10]. Evidence also suggests that complex actions are generated similarly, by means of an organizing hierarchy [11], the general understanding of which proceeds as follows. The prefrontal cortex sits on the top of the action hierarchy and generates an abstract goal-directed action plan. The next level in the hierarchy is composed of the supplementary motor area (SMA) and the premotor cortex (PC). These are thought to be responsible for generating motor programs for voluntary actions and sensory-guided actions, respectively. These areas then send signals to the next lower level, the primary motor cortex (M1), where it is believed that primitive motor patterns are generated. M1 then passes patterned motor signals further downward via the pons and cerebellum to the spinal cord, which then sends out detailed motor commands to corresponding muscles, finally initiating physical movements. That said, it must be noted that the complex action generating hierarchy in the human brain is not so simple. Other parts of the human brain are also involved in the generation of complex actions. In particular studies on apraxia caused by cerebral hemorrhage have suggested that the inferior parietal lobe (IPL) is crucial for generating skilled actions like tool usage [12], [13]. This is because skilled actions, such as manipulating an object as a tool, require motor related multimodal sensory feedback, and this visuo-tactile-proprioceptive integration is developed in the IPL through dense interactions between the frontal and the parietal lobes [14].

Considering the evidence that human brains achieve compositionality through a functional hierarchy embedded in neuronal networks connecting different local regions, our original question returns: How exactly is compositionality realized at the neural circuit level? Specifically, we may ask: Do some neuronal circuits behave as if symbols were represented and manipulated, as digital computers do? Some may argue that the discovery of "grandmother cells" [15] suggests something like symbolic representation in neuronal circuits. However, the evidence does not wholly support such an argument. Some cells do appear to demonstrate quite narrow response selectivity as if activated only by a particular perceptual stimulus, such as one's grandmother's face. However, when tested with diverse visual stimuli [16], it has been found that these same cells can also be activated by other types of visual images. Rather than each item being discretely represented by a corresponding cell, meanings and concepts are more likely encoded in distributed activities of neuronal ensembles. And, if such a distributed representation is the reality in biological brains, how can we imagine compositionality being realized by them?

In pursuing this question, the current paper presumes the following model and from this basis makes two central hypotheses. The model, on one hand, presumes a top-down intentional pathway by which compositional images and plans for acting in and on the world are proactively generated under particular intentional states. On the other hand, the model presumes a bottom-up pathway which is responsible for recognition of the continuously updated perceptual reality. The core hypothesis is that compositionality develops by means of the self-organization of a particular class of dynamic structures. These structures provide for dense interactions between the aforementioned two pathways, both during the course of consolidative learning of perceptual experience and through diverse intentional interactions of the agent with the outer world. It is further speculated that compositionality developed in such neuro-dynamic systems can be naturally grounded in sensory-motor reality. These hypotheses are examined via synthetic modeling studies accompanied by robotics experiments. The robotics experiments include learning of compositional action generation and associative learning of proto-language and corresponding actions. Although these synthetic modeling studies may not be able to prove the biological reality of the proposed hypotheses, it can provide proof of principle for "brain-like" realization of cognitive competence in artifacts, i.e. cognitive robots. The next section describes the basic framework that underlies all subsequent demonstrations.

#### **II. Basic Frameworks of the Models**

This section describes how the top-down proactive intentional processes and the bottom-up recognition processes could be realized in a particular neuro-dynamic system, and how these two processes can interact densely in the course of learning as well as in generating compositional actions. Furthermore, it describes how the model can be extended to produce a functional hierarchy for action generation.

## A. Learning to Predict/Recognize Perceptual Sequences

Tani and colleagues [17]-[19] have shown that learning, generating, and recognizing sensory-motor patterns can be accomplished by extending conventional RNN models, in terms of prediction error minimization within a particular dynamic neural network model, a recurrent neural network with parametric biases (RNNPB), [20], [21]. The dynamics of the model can be described by a difference equation (1):

$$(Y_{t+1}, X_{t+1}) = f(Y_t, X_t, W, \xi)$$
(1)

where  $X_t$  is the current internal neural state, W is a set of learnable parameters such as connectivity weights and biases,  $\xi$  is the intention state, and  $Y_t$  is the perceptual state which is an observable state. This equation represents the top-down intentional process where  $Y_{t+1}$  (the perceptual state at the next time step for a given intentional state  $\xi$ ) is predicted by means of the dynamic mapping from the current internal state  $X_t$  and the current perceptual state  $Y_t$ . This mapping is parameterized by W which is acquired through the learning process described later. This forward dynamics model can be regarded as a generative model with the intentional state being key to regenerate the corresponding perceptual sequence that is learned. Fig. 1(a) illustrates how this idea can be realized in the RNNPB model where the PB units in the input layer represent the intentional state in question. It is noted that  $X_0$  is the initial internal state, set with a neutral value as in the case of learning described next.

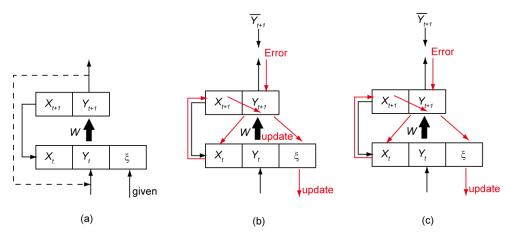


Fig.1. RNNPB model, (a) Perceptual sequences for given intentional states are predicted, (b) Target perceptual sequences are learned by inferring optimal connectivity weights and intentional states by means of prediction error minimization where red arrows denote error signal back-propagation, (c) intentional states for given perceptual sequences are inversely computed by means of prediction error minimization.

The system predicts perceptual sequences to be caused by the intentional state either in an open loop mode or in a closed loop mode. In the open loop mode, an actual movement is made by generating the one-step prediction of the next perceptual state through the use of the current one. In the closed loop mode, a look-ahead prediction of multiple steps of a perceptual sequence is generated by feeding back the predicted perceptual state from the previous step to the current inputs. This process can generate a motor imagery sequence evolving in correspondence with a specific intentional state. Although the RNNPB model employs a hidden units layer between the input and the output layers, these are not shown in Fig. 1.

Next, the learning process is described. The learning of a set of intentional actions in a supervised manner is formulated as the process of minimizing the prediction error between the teaching target perceptual sequence  $\overline{Y}_t$  and its prediction  $Y_t$  in (1). This is accomplished by searching for the optimal values of the learnable parameters W common to all target actions to be learned as well as for the corresponding intentional state  $\xi$  for each action (see Fig.1 b), and is implemented by utilizing the error back-propagation through time (BPTT) algorithm [22]. In the learning process, the top-down prediction of a perceptual sequence is unfolded in time with a particular intentional state given as the input. Then, when compared to the target perceptual sequence, an error signal is generated and back-propagated through the internal state loop to the intention state. Thus, through bottom-up recognition, the connectivity

weights and the intentional states are gradually updated. In fact, through iterations of this learning process, dense interaction between the top-down and the bottom-up processes is facilitated, and in this way projected perceptual images originating from the intentional state can be shaped by and grounded in the reality of the "objective world".

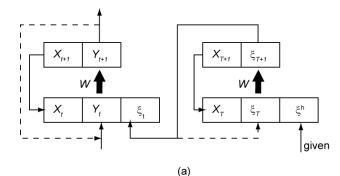
Finally, the process of recognizing a target perceptual sequence can be formulated as a process of searching for an optimal intentional state by which the target sequence can be generated with a minimum error while the learned weight parameters *W* remain fixed (see Fig. 1 (c)). The aforementioned model is formally related to the idea of predictive coding as developed by Rao and Ballard [23]. It also bears formal similarity with that of active inference for which Friston [24] has recently demonstrated a generalized framework under the name of *free-energy minimization*. In the model by Friston [24], the prediction of a subsequent perceptual state is accomplished by means of approximate Bayesian inference through the estimation of both mean and variance.

## **B.** Functional Hierarchy

On the computational view, compositionality requires some hierarchical operations involving the manipulation of a set of elements in a lower level through the application of particular rules from a higher level. This is analogous to storing and retrieving behavior primitives, or words, in the lower level and combining them into goal-directed actions or sentences by following scenarios or plans from the higher level. How can dynamic neural network models represent such mechanisms? One possible way is to consider a cascade of aforementioned RNNPB models operating according to different timescales [18]. Such a model, operative on two levels, can be described as (2):

$$\begin{cases} (Y_{t+1}, X_{t+1}) = f^{l}(Y_{t}, X_{t}, W^{l}, \xi_{t}) \\ (\xi_{T+1}, X_{T+1}) = f^{h}(\xi_{T}, X_{T}, W^{h}, \xi^{h}) \end{cases}$$
(2)

Here, *t* denotes time on the faster timescale with a smaller  $\Delta t$  for each time step in the lower level, and *T* denotes time on the slower timescale with a larger  $\Delta t$  in the higher level, while *l* and *h* denote suffixes for the lower level and the higher level, respectively. The main idea is that  $\xi_t$  in the lower-level dynamic function works as a slowly changing parameter for  $f^l$  which is predicted as  $\xi_{T+1}$  by the higher-level dynamic system operating in the slower timescale (see Fig. 2 (a)).



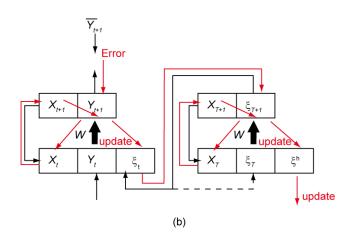


Fig.2. Hierarchically-organized RNNPB model. (a) A perceptual sequence is predicted for intentional states given in the higher level. (b) Target perceptual sequences are either learned or recognized by inferring both connectivity weights and intentional states, or the intentional states only, where red arrows denote the error signal back-propagated from the lower level to the higher level.

The prediction of  $\xi_{T+1}$  is generated as corresponding to the higher-level intentional state  $\xi^h$ . This is an important aspect of the implicit generative model, due to the fundamental role played by the separation of temporal scales. This separation enables higher-level dynamics to contextualize (through the provision of parametric biases) faster fluctuations at lower levels, and recapitulates the causal structure of the (hierarchical) world that the agent is trying to predict. In sum, a functional hierarchy is achieved by means of an "abstract" low-dimensional parametric control over the lower-level dynamic function by the higher-level one. In the learning process, the delta error signal is back-propagated from the lower level to the higher one by being passed from  $\xi_t$  to  $\xi_T$  and finally to the higher-level intentional state  $\xi^h$  whereby parameters  $W^l$  and  $W^h$  as well as  $\xi^h$  are updated in the direction of minimizing error (Fig. 2 (b)). A target perceptual sequence can be recognized simply by updating  $\xi^h$  by

utilizing the delta error delivered from the prediction error for the perceptual inputs.

The aforementioned hierarchical model can be improved, further, by considering that perceptual sequence patterns are experienced as continuous flow rather than as discrete events, and by adjusting the model to operate in continuous time accordingly. For this purpose, consider a continuous time recurrent neural network model (CTRNN) [25], [26] consisting of sub-networks the dynamics of which are characterized by the different values of time constants assigned. Such a model is referred to as a multiple timescales recurrent neural network (MTRNN) [27], [28] in which the forward dynamics of each neural unit is described as (3):

$$\begin{cases} \tau_{i}\dot{u}_{i} = -u_{i} + \sum_{j} w_{ij}a_{j} + \sum_{k} w_{ik}I_{k} \\ y_{i} = \frac{\exp(u_{i})}{\sum_{j \in Out} \exp(u_{j})}, \text{ if } i \in Out \\ a_{i} = \frac{1}{1 + e^{-(u_{i} + b_{i})}}, \text{ otherwise} \end{cases}$$
(3)

where  $\tau_i$ ,  $u_i$ ,  $y_i$ ,  $a_i$  and  $b_i$  are the time constant, the membrane potential, the neural activation state for an output unit and for an internal (context) unit, and the bias of the *i*th unit, respectively. It is important to note, here, that the activation of output units follows a softmax function with  $u_i$  as the potential value at the *i*th cell within *OutN* cells, while the activation of internal units follows a standard sigmoidal function. This treatment is stipulated in order to make the output patterns activate only sparsely, i.e. the summation of *OutN* output units becomes 1.0.  $I_k$  is the *k*th input, and  $w_{ij}$  is the connectivity weight from the *j*th unit to the *i*th unit. When the time constant  $\tau_i$  is set at a larger value, the activation dynamics of the unit tend to be slower. Conversely, with a smaller value, they become faster. If the whole network is built as a cascade of sub-networks, with the higher-level network consisting of dynamic units with a larger time constant, and with the lower-level network with a smaller time constant as illustrated in Fig. 3, a functional hierarchy similar to the one described for the RNNPB with discrete time emerges.

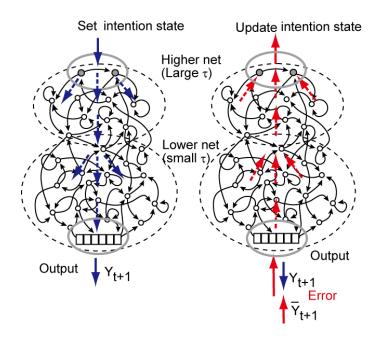


Fig. 3 MTRNN model with top-down perception prediction according to a given intentional state on the left-hand side, and with bottom-up perception recognition and learning on the right-hand side.

On the top-down pathway, the intentional state in the higher-level network is set with particular values for initial states in some neural units, the so-called "intention" units, and slowly changing neural activities in this higher-level network are initiated. This slowly changing activity affects the faster dynamics of the lower-level network by means of parameter modulation and bifurcation, resulting in the modulation of and shifting in sequence patterns predicted by, and generated in, the output units. In the learning process, the error generated in the comparison of the target sequence and the prediction sequence back-propagates through time to the initial states of the intention units in the higher-level network by going through all connectivity weights in the whole network, whereby all of the connectivity weights as well as the initial states for each target sequence are updated.

Through the iterative interactions between top-down prediction and bottom-up error regression, which the whole network undergoes at all levels in a distributed and parallel manner, it is almost inevitable that an adequate functional hierarchy between multiple levels with different timescales can and will self-organize. The exact learning mechanism incorporates a modified BPTT scheme which considers the effects of each time constant assigned to each dynamic unit in MTRNN [27]-[29]. Consider *E* as a summation of prediction errors for all output units and for all time steps, and which is to

be minimized through the learning process. *E* is represented in terms of Kullback–Leibler divergence as:

$$\mathbf{E} = \sum_{t} \sum_{i \in Out} y_{i,t}^* \log\left(\frac{y_{i,t}^*}{y_{i,t}}\right) \tag{4}$$

where  $y_{i,t}^*$  and  $y_{i,t}$  are the target output and output for the *i*th output unit at time step *t*, respectively. Each connectivity weight  $w_{ij}$  is updated in a direction opposite to that of the gradient  $\partial E / \partial w_{ij}$  as:

$$w_{ij}(n+1) = w_{ij}(n) - \alpha \frac{\partial E}{\partial w_{ij}}$$
(5)

where  $\alpha$  is the learning rate and *n* is an index representing the iteration step in the learning process. Then,  $\frac{\partial E}{\partial w_{ij}}$  is given by:

$$\frac{\partial E}{\partial w_{ij}} = \sum_{t} \frac{1}{\tau_i} \frac{\partial E}{\partial u_{i,t}} a_{j,t-1}$$
(6)

and the delta error at the *i*th unit  $\frac{\partial E}{\partial u_{i,t}}$  is recursively calculated from the following formula:

$$\frac{\partial E}{\partial u_{i,t}} = \begin{cases} y_{i,t} - y_{i,t}^* + \left(1 - \frac{1}{\tau_i}\right) \frac{\partial E}{\partial u_{i,t+1}} & i \in Out\\ \sum_{k \in N} \frac{\partial E}{\partial u_{k,t+1}} \left[ \delta_{ik} \left(1 - \frac{1}{\tau_i}\right) + \frac{1}{\tau_k} w_{ki} f'(u_{i,t}) \right] & i \notin Out \end{cases}$$
(7)

where f'() is the derivative of the sigmoid output function and  $\delta_{ik}$  is Kronecker delta function. By looking at the second line of (7) it can be seen that the *i*th unit in the current step *t* inherits a large portion  $\left(1 - \frac{1}{\tau_i}\right)$  of the delta error  $\frac{\partial E}{\partial u_{i,t+1}}$  from the same unit in the next step t+1 when its time constant  $\tau_i$  is relatively large. It is noted that (7) turns out to be the conventional, discrete time version of BPTT with  $\tau_i$  set as 1.0. This means that, in a slow dynamic network with a large time constant, error back-propagates through time with a small decay rate. This enables learning of long-term correlations latent in the target time profiles by filtering out fast changes in the profiles. In addition, an optimal initial state of the intention unit can be searched for by updating the membrane potential state  $u_{i,0}$  and by utilizing the delta error back-propagated through time to the unit of the 0<sup>th</sup> step as:

$$u_{i,0}(n+1) = u_{i,0}(n) - \alpha \frac{\partial E}{\partial u_{i,0}}$$
(8)

In summary, the preceding describes a generic scheme based on the hierarchical composition of (autonomous) dynamical systems that serve as a generative model of both exteroceptive and proprioceptive inputs. The deep hierarchical structure, and the separation of temporal scales described in this model (through descending parametric biases), enable the agent to predict and to learn, informed as it is by the sensorimotor contingencies entailed in its environment (and its motor plant). Crucially, on this point, all levels of the model rest upon continuous dynamics - where symbolic representation is implicit in the (unstable) fixed points of the dynamical system. A second key aspect of this framework is that there are no explicit sensory response links. These are acquired through descending predictions of sensory observations in both the visual and motor domain. This means that sensorimotor constructs are represented as amodal dynamics at the top of the hierarchy (with protracted timescales), providing bilateral corollary discharges or predictions about the state of the world and the motor plant. In what follows, we will use this scheme in a number of experimental contexts to show how it accounts for the learning of motor primitives, imitative behavior, and for the generation of spontaneous yet deterministic behavior, all through the use of (entrained) chaotic dynamics.

#### **III. Neuro-Robotics Experiments**

This section describes robotics experiments utilizing the aforementioned dynamic neural network models in the examination of how an agent may develop the capacity to compose complex cognitive behaviors while remaining grounded in sensory-motor reality. These robotics experiments explore research topics including (a) dynamic shifts of action intentions via recognition of situational changes in the environment, (b) the development of compositional concepts via associative learning between proto-language and behaviors, (c) the tutoring of complex skilled actions, and (d) the spontaneous generation of combinatorial action sequences. Further, analysis of the dynamical structures self-organized in the models under consideration will also shed light on essential neuro-dynamic mechanisms underlying compositionality in human-level cognition.

#### A. Dynamic Shifts of Action Intentions via Bottom-Up and Top-Down Interactions

The following robotics experiments, utilizing the RNNPB model described earlier, examine how the top-down intention to act on the physical environment can be dynamically adapted by the bottom-up recognition of perceptual reality according to situational changes in the environment (for further details, see [30]). In the current task, a small humanoid robot made by the Sony Corporation learned to generate ball "playing" behaviors under human tutoring. The human tutor manually guided the robot's

movements, helping it to manipulate a ball by grasping its arms, while the robot perceived sequences of two types, visual and proprioception. The robot's visual perception  $S_t$  took the form of the 3-dimensional relative position of the ball on the task table as measured by color tracking, and the proprioception  $M_t$  of the bimanual arms and hands was represented by 8 DOFs of the corresponding joint angles. After repeated training, the robot was able to predict the visuo-proprioceptive state of the next step in an action sequence. The actual movement of the robot arms was generated by sending the prediction, the next target values, of the next proprioceptive step in terms of 8 DOFs of joint angles to the motor controllers of the arms. The robot was trained for two types of ball play. "Play-1" consisted in rolling the ball from the left-hand side to the right-hand side, and vice-versa, repeatedly by gently pushing it with both hands. "Play-2" consisted in repeatedly grasping the ball in the middle position, lifting it up, and then dropping it. By following the scheme shown in Fig. 1(b), training of the RNNPB was conducted where the visuo-proprioceptive sequences  $(\overline{S}_t, \overline{M}_t)$  obtained during the tutoring sessions was utilized as target training sequences. The training data consisted of 6 cycles of sequences both for play-1 and play-2. Note that the objective of the training was to make the network regenerate two types of target visuo-proprioceptive sequences depending on the intention states (the PB values) which had been self-determined through the training process. We employed an RNNPB that has 11 input units and 11 prediction output units. It also has two PB units, 50 hidden units, and 70 context units representing the internal state  $X_t$ . The learning was iterated for 50,000 epochs, starting from an initial random set of synaptic weights. The final root-mean-square error of the output units became less than 0.0003. It was assured that a different PB vector value was determined for each type of play.

After the training, the robot's behavior generation for the learned types of ball-play was tested by following a scheme of online generation and recognition. The robot's movements were generated by feeding the next-step predictions of proprioceptive states (the joint angles of bimanual arms and hands) to the position controller of the robot, and the PB vector was updated by means of on-line recognition of visual perception (the perception of ball position). For the online recognition process (PB regression utilizing the prediction error), 50 instances of forward and back-propagation were conducted using a 30-step window on the immediate past in order to determine the PB at each next time step. This repeated behavior generation experiment showed that, although the robot tended to stably generate one of the learned types of ball play, the type of play switched from one to the other intermittently. Fig. 4 shows a sequence of photo snaps and the corresponding plots of time evolution of

parameters during this particular behavior generation.

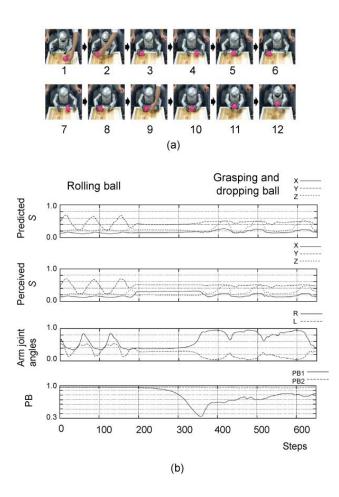


Fig. 4 Autonomous behavioral switching from 'rolling ball' to 'grasping and dropping ball' in the Sony humanoid robot QRIO. (a) The corresponding photo snap sequence. (b) Time profiles for the predicted ball position (X, Y, Z) in vision, its actual perception, predicted joint angles (a representative joint angle for each arm), and the PB vector (PB1, PB2).

In Fig. 4(a) it can be seen that behavior switching takes place in the ninth photo snap. Fig. 4(b) shows time profiles for the predicted ball position, its actual perception, predicted joint angles (representing two out of a total of eight DOFs), and the two-dimensional PB vector. From the time profiles of the ball position and the joint angles, it can be seen that behavior switching took place between 200 and 350 steps and that it was associated with a shift in the PB vector. The behavior switching was initiated by a small fluctuation in ball positioning during "rolling ball" play at around the 180<sup>th</sup> time step when the ball came slightly more toward the center than was predicted in the

case of pushing the ball from the right side. The resultant prediction error caused gradual modulation of the PB vector toward the value for "grasping the ball located at the center" which caused the arm to gradually push the ball toward the center more. Finally, play-2 of "grasping the ball in the center position and dropping it" was initiated by achieving a perfect match between the perceived ball position and the one predicted by the modulated PB value. Here, we can see how intention can be dynamically modulated through the bottom-up recognition of the perceptual reality by utilizing the error regression scheme.

#### **B.** Associative Learning Between Proto-Language and Behaviors

The faculty for language and the faculties for other types of action have been treated independently in conventional neuroscience. Recently, however, some researchers have been looking at these functions utilizing various brain imaging techniques, including fMRI, PET and EEG, and this research has begun to suggest a certain dependency between them. Hauk et al [31] showed in a functional MRI experiment that reading action related words with different end effectors, e.g. "Lick", "Pick" and "Kick", evoke neural activities in motor areas corresponding with the local areas responsible for generating motor movements in the face, arm and leg, respectively. This result, as well as that reported in [32], suggest that understanding words or sentences related to actions may require the presence of specific motor circuits responsible for generating those actions, and therefore the parts of the brain responsible for language use and other forms of action might be interdependent. Pulvemuller [33] argues that, if everyday experiences of speech and corresponding sensory-motor signals tend to overlap during infant development, synaptic connectivity between the two circuits can be reinforced through Hebbian learning. This suggests a possibility that meaning and concepts of words and sentences are acquired as associations with related sensory-motor experiences, as discussed in the usage-based approach by Tomasello [34], i.e. cognitive linguistics.

The robotics experiment described in this section explores this possibility, that the so-called "semantically combinatorial language of thought" as mentioned by Fodor and Pylyshyn [35] can be developed in terms of neuro-dynamic structures provided that dense interactions are allowed between linguistic processes and behavioral ones. In this robotics experiment, we examined how a set of simple sentences consisting of verbs and object nouns can be understood, and the corresponding actions produced, by robots utilizing an extended RNNPB model [36]. The model consists of a linguistic RNNPB and a behavioral RNNPB interconnected through PB units.

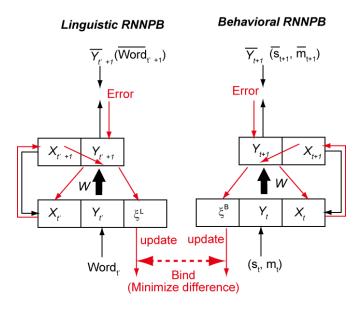


Fig. 5. A linguistic RNNPB predicting word sequences and a behavioral RNNPB predicting sensory-motor sequences, interconnected by PB units, where the PB vector of each RNNPB is updated by utilizing its own error signal while minimizing the difference between two.

The central idea behind the model was that the PB activation vectors in both modules should be bound in order to become identical for generating pairs of corresponding linguistic and behavioral sequences via learning (see Fig. 5). In other words, stimulus response links are not formed by simply associating, or mapping, between words and actions, but by constructing amodal, high level, dynamical representations that are both sensory and motor in nature. In what follows, we will show that this representational space embodies the fundamental distinction between the different actions that can be undertaken and the objects that are the targets of those actions. More specifically, in the course of the bound learning of pairs of linguistic and behavioral sequences, the PB activation vectors in both modules were updated in the direction of minimizing their differences as well as in the direction of minimizing the prediction error in either modality, alone. By passing the error signals back-propagated from both modules to the shared PB units, a sort of unified representation between the two modalities could be formed through the self-organization of the PB activations. After the learning converged for all of the pairs, the capacity to understand sentences was tested as follows. A particular word sequence was shown to the linguistic module as a target to be recognized by the PB regression scheme. Then, the PB value obtained as the result of regenerating the word sequence with the minimum error was used to activate the

behavioral RNNPB in order to generate a prediction of the corresponding perceptual sequence responsible for generating that particular robot behavior. Here, a tempting expectation was that compositionality hidden in the perceived data of the linguistic modality and behavioral modality could be captured in the internal neuro-dynamic structures shared by these two modalities via consolidative learning accompanied by the top-down and the bottom-up interactions. This hypothesis was evaluated through experiments utilizing a physically mobile robot.

A mobile robot equipped with a camera and one DOF arm was placed in a workspace where red, blue and green objects were always located to the left, center, and right of the robot, respectively (Fig. 6(a)).

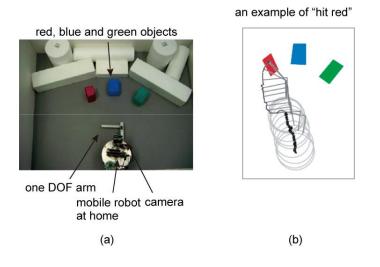


Fig. 6. The mobile robot with video camera and arm used in the experiment for proto-language and behavior association learning. (a) The robot is facing red, blue and green objects at its home position. (b) An example of a behavioral trajectory for hitting the red object.

We considered a set of imperative sentences consisting of 3 verbs (point, push, hit) and 6 object nouns (left, center, right, red, blue, green). In these sentences, "point blue" indicated that the robot had to point to the blue object by extending its arm, "push red" indicated that the robot had to move to the red object and push it with its body, and "hit left" indicated that the robot had to move to the object to its left and hit that object with its arm. Note that "red" and "left" were synonymous in the employed workspace setting, as were "blue" and "center" and "green" and "right". For each given combination of verb and noun, corresponding actions in terms of perceptual sequences consisting of more than 100 steps for each trial were tutored by guiding the robot with a remote

controller, while slight variances in object positions as well as in the robot's starting positions were introduced during each trial of tutoring. Such tutoring was repeated three times for each imperative sentence. For the purpose of investigating the capacity to generalize the learning, only 14 out of 18 possible sentences were trained and also bound to the corresponding behavioral sequences tutored. The remaining 4 behavioral sequences were trained without being bound to the linguistic sentences. The behavioral RNNPB received 26-dimensional multimodal perceptual inputs and it output their prediction for each next step. The multimodal perceptions included 21 feature values for encoding visual images, along with 2 measured torque values (an average torque value of both wheels, and a torque value of the arm), the velocities of the two motor wheels and a joint angle for the one DOF arm as proprioception. The behavioral RNNPB had 70 hidden units, 4 context units and 6 PB units. The linguistic RNNPB received a single word at each step which was encoded by 10 input units (each unit uniquely represents a single world out of 9 words and a starting symbol) and output a prediction of the next word as encoded in 9 output units in the same manner. This RNNPB had 50 hidden units, 4 context units and 6 PB units. It must be noted that time-step processing by the forward dynamics of these two RNNPB modules was not necessarily synchronous, as the learning of both modules was conducted off-line and behavior generation was preceded by the recognition of imperative sentences.

The learning process converged with a root mean square error of 0.0091 for the linguistic module and 0.025 for the behavior module after 50000 epochs of iterative training of the whole network. In the succeeding behavior generation test, it was found that the robot could generate correct actions for all 18 sentences. Crucially, these included the four untrained sentences. In other words, the agent was able to generalize the abstract and cognitive structure of its world as evident in its responses to novel contexts. An example of a "hit red" trajectory generated by the robot is shown in Fig. 6(b). It was also found that the robot could generate the corresponding behaviors quite robustly against miscellaneous perturbations. For example, in the cases of "hitting" or "pushing" object behaviors, the robot could continue to track the target object even after the object was slightly moved while approaching. A such example movie can be seen in video 1 in the on-line supplementary material provided in [37]. We examined how each sentence was mapped to the PB vector space. Fig. 7 shows this sentence mapping to the PB space with its two principal components.

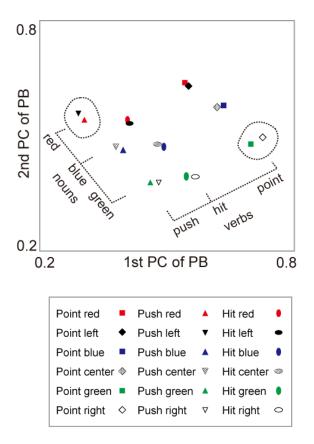


Fig. 7. The observed mapping of 18 imperative sentences into the PB space constructed by its  $1^{st}$  and  $2^{nd}$  principal components. The four PB points surrounded by dotted circles represent PB values for untrained sentences.

Observe that the mapping appears with a 2-dimensional grid structure with one dimension for verbs and the other for nouns, where all sentences with the same verbs followed by synonymous nouns appeared close in the space. It is noted that even sentences of the unlearned combinations, "push red|left" and "point green|right" were mapped to adequate positions in the grid (indicated by dotted circles). And, following further experiments, it appears that these untrained sentences were recognized correctly because their meanings became inferable due to structural relations with learned ones. The same experiment was repeated 3 times, each time introducing different sets of 4 unbound sentences. We found that the same generalization was attained by the trained network model in every case where a similar two-dimensional grid structure was formed in the PB mapping. However, it was also observed that this generalization was lost when more than four sentences were removed from the bound learning. These results imply that meanings could be acquired as a relational structure among many of the sentences, and that such structures can be developed in the distributed activation

patterns of neural units as the result of the generalization of learning if an only if a sufficient number of exemplars is provided.

## C. Learning Compositional Actions via Self-Organization of Functional Hierarchy

This robotics experiment examined how a functional hierarchy can be developed in the course of learning complex actions dealing with object manipulation by utilizing the previously described MTRNN model. As mentioned in the introductory section, it is generally considered that complex, goal-directed actions can be generated by combining reusable primitives. A difficult question arises, however, concerning how behavior primitives can be extracted from direct experience and then be stored in the memory pool, when the perceptual sequence itself is naively experienced as unarticulated flow, without explicit cues guiding segmentation into those behavior primitives. Another question, related to this one, concerns how those primitives, once stored in memory, can be recombined to generate smooth and continuous patterns of behavior, complex yet fluid operations which Luria [38] metaphorically refers to as "kinetic melodies." The central problem here is that cognitive competency for compositional action generation seems to require two incompatible aspects. On the one hand, there appear to be algebraic operations on behavior primitives treated as if they were discrete, concrete objects, and on the other hand there is the fluid and context sensitive concatenation of one primitive with another matching the delicate flow of perceptual experience. The following humanoid robotics experiment focused on this issue.

The experiment was conducted with the Sony humanoid robot platform utilizing the MTRNN architecture shown in Fig. 8 [27].

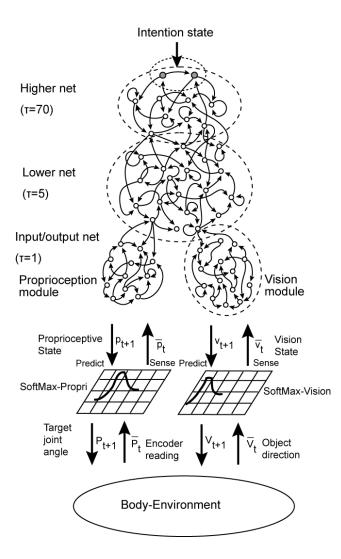


Fig. 8. The MTRNN architecture utilized in humanoid robot experiments examining object manipulation.

By implementing a color-based object-tracking camera on the robot's head, two-dimensional camera head angles targeting a red colored object represented the visually perceived object position  $V_t$ . Eight-dimensional joint angles in bimanual arms represented the proprioceptive state  $P_t$ . The visual state  $v_t$  and the proprioception  $m_t$ were mapped to softmax activation patterns of 36 cells for visual state  $v_t$  and 64 cells for proprioceptive state  $p_t$  by utilizing corresponding topology-preserving maps implemented in Kohonen networks [39]. The current visual state  $v_t$  and proprioceptive state  $p_t$  were fed into the input units of vision and the proprioception input-output networks, respectively, in order to predict its state at each next time step in the output units. The whole MTRNN architecture consisted of a higher-level network containing

20 slow context units ( $\tau_s = 70$ ), a lower-level network containing 30 fast context units  $(\tau_f = 5)$ , the vision input-output network containing 36 units  $(\tau_v = 2)$  and the proprioception input-output network containing 64 input-output units ( $\tau_p = 2$ ). Two slow context units in the higher-level network were assigned to represent intentional states in terms of the initial states. The units within each network, the higher-level network and the lower-level network, were fully interconnected, as were the units within both the vision and proprioception networks. However, neither were the units in the higher-level network connected directly with the units in the input-output network, nor were the units in the vision network connected directly with the units in the proprioception network. Our assumption was that this kind of constraint on the network connectivity would allow for the development of information bottlenecks and hubs in the lower-level network. Starting with a particular initial state (representing intention), and following the forward dynamics of the whole network, the network predicted the vision state and the proprioceptive state of each next time step by receiving the values from the current time step from the corresponding perception channels. The prediction of the proprioception state at each next time step was sent to the PID controller of the robot in order to generate the appropriate motor command for each joint motor to achieve the predicted posture of the robot at that next time step.

The robot was trained to generate five different task behaviors with an object under the physical guidance of a human tutor as described in previous experiments. The goal of each task behavior was to generate a different sequence of behavior primitives in order to manipulate the object in different ways, such as reaching for the object, moving the object up and down (UD), left and right (LR) and forward and backward (FB) a specific number of repetitions. There was one behavior primitive, clapping hands (CL), which did not involve with the object. All five task behaviors started from the home position and ended by returning to that same position (see Fig. 9).

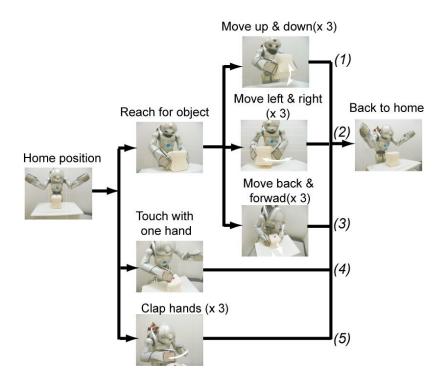


Fig. 9. A humanoid robot made by Sony is trained for five behavioral tasks, each of which is composed of a sequence of behavior primitives.

The tutor taught the robot each of these task behaviors, changing the object position five times (2cm each) between the left end and the right end in the task table, except for task 5 which did not contain object-related behavior. It is important to note, here, that no explicit cues were provided for the segmentation of the tutored visuo-proprioceptive sequences into behavior primitives.

Training of the MTRNN was completed with 5000 iterations of BPTT for each of the tutored sequences. This resulted in a root mean square error of 0.009. The robot's performance was tested for all five task behaviors by changing the initial object position five times within the trained range. It was shown that the robot performed all task behaviors successfully. Here, "success" means that the robot could generate specific patterns within specific ranges of movement amplitudes for UD, LR, FB and CL, for a specific number of repetitions, without dropping the object (for more details, see [27].) It is important to note, also, that this same learning experiment was repeated five times, in each case with similar results obtained with near perfect behavior regenerations. The robot failed in only one trial out of 25 trials by dropping the object while grasping it.

Fig. 10 illustrates representative task regenerations, showing the development over time of essential system variables, with task-2 and task-5 represented in the

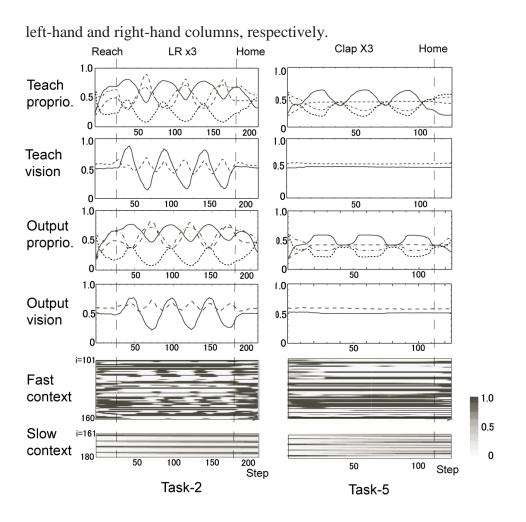


Fig. 10. Examples of teaching patterns and generated patterns for task-2 and task-5. Proprioception and vision trajectories during teaching (in the first and the second rows) and during generation (in the third and the forth rows) are shown. The activation patterns for 60 fast context units and for 20 slow context units are shown in the 6<sup>th</sup> and 7<sup>th</sup> rows, respectively.

Each plot shows target signals for the four representative dimensions of proprioception and two-dimensional visual information for object position during tutoring, with corresponding generated outputs, and with the activations of 60 fast context units and 20 slow context units in grey-scale representation. Looking at the activation dynamics of the fast context units in the lower network, it is clear that their dynamics were correlated with visuo-proprioceptive trajectories. On the other hand, the activation patterns in the slow context units changed much more slowly. From this observation, a hypothesis can be drawn, that a sequence of primitive patterns, embedded in the lower sub-network and characterized by fast dynamics, was learned in the higher sub-network and characterized by slow dynamics.

According to this hypothesis and assumed functional roles of the slow and fast dynamics in the model network, one would anticipate that novel combinations of primitives would be generated only by modulating the activity of the slow context units. In order to test this idea, the network was retrained to generate additional, novel behavior sequences, which were to be assembled from new combinations of prior-learned primitives. Most importantly, during this additional training, only the connectivity weights in the higher network were allowed to change, not the ones in the lower and the input-output networks. The additional training consisted of two additional tasks. In task-6, the robot was required to move the object up and down three times, then to move the object left and right three times, and finally to go back to the home position. In task-7, the robot was required to move the object backward and forward three times, then to touch the object with one hand, and finally to go back to the home position. After the retraining, the robot reproduced the novel behavior sequences successfully with generalization across object locations (an example robot movie can be seen in video 2 in the supplementary material in [37].) Fig. 11 displays an example of regeneration of the novel behavior sequence of task-6, in which we can see that the visuo-proprioceptive trajectories were perfectly generated as compared to the teaching target ones. It can be also seen that activation patterns of the fast context units synchronized with the perceptual sequences, whereas those for the slow context units changed much more slowly, in accordance with the earlier experimental design.

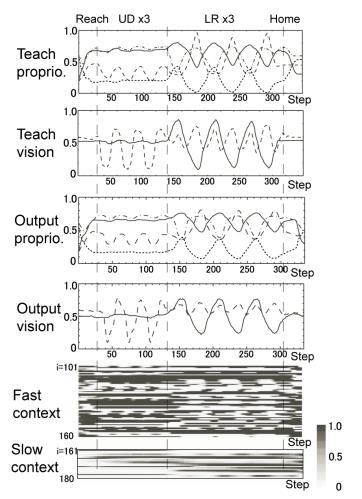


Fig. 11. An example of teaching and generated patterns for the novel behavior sequence of task-6.

In order to examine the effects on the learning process of timescale differences between the slow network and the fast one, the experiment that included training of the additional novel sequences was further extended. In this extension, the difference in the timescales was described in terms of the ratio of  $\tau$  values in the fast and slow context units as  $(\tau_s/\tau_f)$ . First, after initially randomizing all the connectivity weights in the whole network in each instance, the basic sequences were trained five times, with the time constant ratio  $(\tau_s/\tau_f)$  set to five different values from 1.0 to 14.0, and with  $\tau_f$  fixed at 5.0. Then, the additional novel sequences were trained five times. In these instances, the connectivity weights were randomized only in the higher network while the weights in other sub-networks were preserved, and while the time constant ratio  $(\tau_s/\tau_f)$  was adjusted in the same way. In both training cases, learning was iterated for 5000 epochs. As a result, the average root mean square error achieved over five trials for five different time constant ratios is shown for both basic cases and additional cases in Fig.

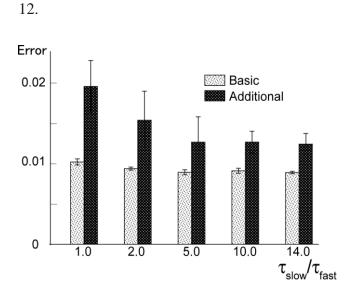


Fig. 12. The average root mean square error over five training trials, each with a different time constant ratio, is shown for both the basic learning case and the additional one.

It can be seen that the learning error for the additional training case started to increase significantly when the time constant ratio was set to less than 5.0, while the one for the basic case remained almost constant regardless of the time constant ratio. It was also found that the robot could not generate both of the novel behavior sequences successfully when those sequences were trained with time constant ratio settings of 2.0 or 1.0. From these results, it can be inferred that the higher network could not reorganize the novel primitive sequences simply by adapting the weights in the higher network, because the primitives had not been acquired in the lower network in a reusable manner through the basic training phase when the time constant ratio was set too small. This is because two types of memories, one for primitive patterns and the other for the sequencing of them, cannot be segregated in the lower and the higher networks in the case of learning under the condition of the small time constant ratio. On the other hand, with larger time constant ratio values, such as 5.0 and 14.0, it is postulated that the functional hierarchy self-organized between the two sub-networks such that a set of primitive patterns stored in the lower network could be reutilized to generate the novel sequences of the primitives in the higher one.

It was observed that the time profiles of the slow context activities were smooth and uncorrelated with detail profiles either in the visuo-proprioceptive sequences or in the fast context unit activation sequences. Although the profiles of those slow context activities drastically changed as the primitives were shifted from one to another, they never showed stepwise changes at the moments that primitives were switched, but rather only showed continuous, smooth changes. Most importantly, it can thus be said that the higher-level sequencing was realized not in terms of discrete event-like sequences, but in terms of the continuous flow of collective neural activities. The exact profiles of the slow context activities should be determined by the consolidative learning processes which attempt to find compromise between two potentially conflicting factors. One of these factors is that the slow context unit activities can change only gradually because of their time constant constraint, and the other is that the prediction output error in the lower-level network should be minimized by adequately modulating the slow context activities which work as nonlinear parameters, regulating the lower-level network dynamics.

The gradually changing profiles in the slow context units seem to contain some contextual information, useful for counting cycle times in cyclic patterns as well as for predicting the next primitives to switch. When the activities in the slow context units were slightly perturbed by adding artificial noises, the counting became imprecise with plus or minus 1. However, it was always observed that these behaviors were smoothly connected to the next primitive and that transitions to the next primitive never took place mid-way through an ongoing primitive. For example, in task-6, moving the object up and down an incorrect number of repetitions (four times rather than the correct three times as had been taught), smoothly connected to the next primitive of moving the object to the left and right after locating the object on the floor, even though the cycle times were counted inaccurately. This implies that what we may call "fluid compositionality" had been developed via iterative interactions between the higher-level neural dynamics and the lower level during the learning process.

# **D.** Learning to Generate Spontaneous Combinations of Primitive Actions

The previous experiment showed how robots can learn to generate a set of task behaviors, each of which comprising of a deterministic sequential combination of behavior primitives. However, everyday behaviors of human beings appear to be spontaneous, their sequences being not so fully predetermined. Consider the actions involved in making pasta as an example. After pouring water into a pot, and putting it on a gas stove, I can either light the gas stove or put a spoonful of salt in the water. Or, very often, I completely forget to add salt during the preparation of the pasta, only to remember the salt later when I take the first bite. As this example illustrates, some segments of action sequences are deterministic, and must be done in order to satisfy the goal, while others are nondeterministic, even optional, and where we might find spontaneity in action generation.

Psychologists have studied this aspect of human behavior in terms of statistical learning. Their observations on child development as well as adult learning have suggested that chunk structures can be extracted through statistical learning with a sufficiently large number of perceptual and behavioral experiences [40]-[42]. Here, the term "chunk structures" denotes repeatable sequence patterns as chunks, or primitives, and takes account of probabilistic state transitions between those chunks or primitives. One important question, here, is how dynamic neural network models can learn to generate "spontaneous behaviors" by extracting the aforementioned statistical structures for chunking. And furthermore, what is the origin of the probability underlying the statistical structure observed in spontaneous behaviors?

For the purpose of investigating these questions, we conducted the following robot learning experiment involving statistical learning of primitive action transition sequences [29]. The same humanoid robot with the same setting described in the previous section was trained to imitate object manipulation behaviors though direct guidance by a tutor. The target task to be tutored contained stochastic transitions between primitive actions as shown in Fig. 13.

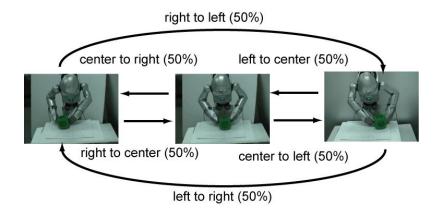


Fig. 13. Task of successive stochastic transitions of action primitives tutored for a Sony-made humanoid robot. Each action primitive starts with both hands approaching a green object, grasping and moving the object to two alternative possible positions with 50% probability for each, releasing the object, and finally ends with the hands going back to the home position.

A colored object was placed on a table in one of three positions (left, center, or right),

and the tutor repeated a primitive action proceeding as follows. Beginning from the home position, both hands approached the object for grasping, then grasped the object. Then, the object was moved to one of two possible positions with equal probability (50%). After releasing the object, the hands returned to the home position. The tutoring of the action was performed continuously, with each next object position determined randomly, and no explicit cues were provided for segmenting the sequences. This tutoring process generated 24 training sequences, each of which consisted of 20 transitions of primitive actions, amounting to roughly 2500 time steps of continuous visuo-proprioceptive sequences. Using the same scheme described in the previous section, this experiment used these training sequences for off-line training of the MTRNN. The MTRNN consisted of a higher-level network containing 30 context units ( $\tau_f = 20$ ), and an input-output network containing 16 gated modular networks with each of which comprised of 10 neural units ( $\tau_{io} = 2$ ) (for further details see [29]).

After the training of the network, we tested the robot for its ability to imitate each tutored sequence. Beginning with the network set with the acquired initial state, the trained primitive action sequences consisting of several primitive action transitions were reproduced exactly during the initial period. However, generated sequences gradually started to deviate from the learned ones. Newly generated sequences deviating from the learned ones were aperiodic, with various sequential combinations of moving the object to left, center or right being observed. Statistical analysis conducted on the transition sequences generated over longer periods (300 transitions of action primitives) showed that the probability of transitioning to one of two possible alternatives was in the range of 40% to 60% for each position, approximating that of the tutored sequences. An example movie for demonstrating the corresponding robot behaviors can be seen in video 3 in the supplementary material [37].

In order to more rigorously examine the capability of the model network to extract stochastic structures hidden in the tutored sequences, we performed analyses on the look-ahead prediction sequences generated by the model network during its closed-loop operation while repeating the training of the network under different conditions. In the closed-loop operation, long sequences of look-ahead prediction for the visuo-proprioceptive state in terms of "motor imagery" can be generated by feeding the current prediction outputs to the next inputs without using the actual sensory inputs, as described previously. Fig. 14 shows an example of the closed-loop generation by the prior trained network, in which the neural activation sequences in units with different timescales on different network levels, and the associated visuo-proprioceptive

sequences, can be seen.

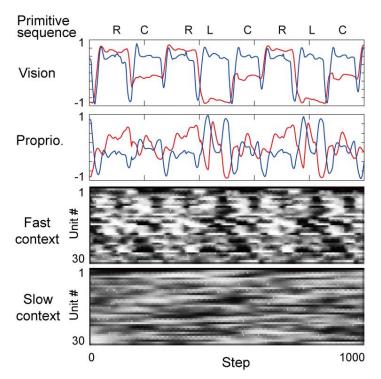


Fig. 14. Forward dynamics generated by the closed-loop operation of a trained network showing a time series of primitive action labels (R, C and L) in terms of vision (relative object position in two dimensions), proprioception (two representative dimensions), and activities of 30 fast context units and 30 slow context units with grayscale plots.

This figure demonstrates that neural activities in the lower-level network and in the higher-level network developed with their intrinsic timescale dynamics, as had been observed in the previous experiment using the MTRNN.

Next, an analysis was conducted for cases introducing different transition probabilities in the tutoring. For this purpose, the tutoring sequences were newly generated by changing the transition probability (the probability of selecting an action of "Right to Center" in Fig. 14) from the original 50% to 25% and to 12.5%. The number of generated tutoring sequences remained the same as during the previous tutoring, one for each transition probability case. The network training was repeated 100 times, utilizing different random settings for the initial weights in each case. Then the transition probability was measured for the visuo-proprioceptive sequences produced via the closed-loop operation of each trained network. Fig. 15 shows the mean of the transition probability obtained from the results of 100 trials of learning for each of the

aforementioned three probabilities. It can be seen that the transition probabilities of the reproduced actions mostly followed the target ones.

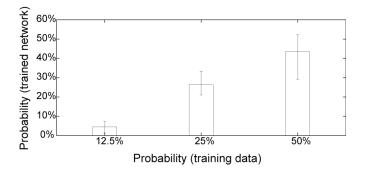


Fig. 15. The mean probability of selecting the action "Right to Center" of 100 trained networks for three different probabilities (12.5%, 25% and 50%) during the tutoring session.

This result implies that the employed model could learn to extract the statistical structures of chunking with their corresponding transition probabilities from the tutored sequences.

Next, we investigated the main issue, namely the origin of indeterminacy or spontaneity in choosing action primitives. In order to avoid the possibility that the stochastic property was originating from miscellaneous real world noise sources, including sensory noise and mechanical noise, the intrinsic dynamics of the network model attained by the closed-loop operation was analyzed again. For purposes of examining the dynamic characteristics of the network quantitatively, a dynamic measure known as the Lyapunov exponent was calculated for the activity of each sub-network during generation of motor imagery. A positive or negative Lyapunov exponent value indicates the rate of divergence or convergence of adjacent trajectories in a given dynamic system, respectively. A positive value for the maximum Lyapunov exponent (MLE) as the largest component indicates that chaos is generated in the system. We computed the MLEs for the higher- and the lower-level networks (see APENDIX-A for the method.) The computation was repeated 100 times with different connectivity weights developed from the initial weights randomized with different seeds but under the same learning conditions. The computation results showed that the average MLE for 100 trials of training was 0.000533 for the higher-level network and -0.007424 for the lower-level network. It was also shown that the probability of showing a positive value of the MLE was 94% for the higher-level network and 0% for the lower-level network.

The result indicates that deterministic chaos was formed in the higher-level network. Results from an additional experiment on artificial lesions in the network model agree with this indication. When the higher-level network was *lesioned* by forcing activation values of all the context units in the network to a constant value, only simple, periodic sequences of action primitive transitions appeared instead of pseudo-stochastic sequences of them. It can be inferred from these results that deterministic chaos developed in the higher level can generate pseudo-stochastic transitioning between action primitives stored in the lower-level network.

For the purpose of investigating possible contributions of the time constant ratio ( $\tau_s/\tau_f$ ) to the formation of chaos in the sub-networks, the same computation was repeated while reducing  $\tau_s$  from the original value of 100 to 50, and to 20, while  $\tau_f$  was fixed at 20. The results showed that the average MLE of the higher-level network became negative, -0.000068 for  $\tau_s$  set at 50, and -0.005069 for  $\tau_s$  set at 20. The average MLE of the lower-level network was found to also be negative for both of these higher-level network time constant conditions. When the time constant ratio was reduced to less than or equal to half of the original, the MLE of both networks became negative. These results indicate that deterministic chaos can be generated in the higher-level network as a consequence of the learning processes, provided that the time constant of the higher-level network is set to an adequately large value as compared to that of the lower-level network.

In summary, the current investigation showed that the MTRNN can generate "spontaneous behaviors" by self-organizing chaos in the higher-level network as the results of statistical learning of perceptual flow. Our analysis confirmed that the chaotic dynamics incorporated with the lower-level network dynamics can realize spontaneous selection of primitive actions by reflecting the observed statistical structures of chunking.

#### **IV. Discussion and Conclusion**

The current paper began with questions concerning how the compositionality assumed in human brains can be developed while remaining grounded in sensory-motor reality. It was conjectured that this compositionality can be developed as embedded in particular neuro-dynamic systems through dense interactions between top-down intentional processes and the bottom-up recognition of the perceptual reality, with such compositional structures remaining naturally grounded. Testing these conjectures, the current paper proposed specific dynamic neural network models, including the RNNPB and the MTRNN, for controlling sensory-motor systems realized under the principle of prediction error minimization. In the process of minimizing the error, such dynamic neural networks learned to predict the perceptual outcomes for particular intention states by forward computation, and they also learned to recognize the perceptual reality by inversely searching corresponding intentional states. It was further speculated that these dynamic neural networks could be incorporated with a "functional hierarchy" such as that observed in human brains by utilizing timescale differences assumed between the sub-networks.

A set of robotics experiments was conducted, examining the aforementioned conjectures. The first robotics experiment, utilizing the RNNPB, showed that the intention of a robot could be dynamically shifted, adapting to sudden situation changes in the environment by means of the regression of possible error between the top-down prediction and the bottom-recognition of the perceptual outcome. What we witnessed was an emergence of circular causality in embodied cognition as has been discussed by Freeman [43], where an intention generates an action, which causes a perception in the environment, which causes a dynamic shift in the intention, which generates another action. The second robotics experiment examined how concepts for actions could be acquired through dynamic interactions between a linguistic module and a behavior module, both of which being implemented by RNNPB models. Through interactive learning between the two modules, by utilizing the error information back-propagated from each of them, the results of this experiment indicated that a compositional structure for expressing combinations of verbs and nouns can be developed with generalization in the distributed representations self-organized in PB activities.

The third experiment, using the MTRNN model, showed that a functional hierarchy can be developed in the model network through iterative learning of the continuous perceptual flow, where a set of reusable behavior primitives was developed in the lower-level network and where the sequencing of these was realized in the higher-level network provided that the time constant parameter of each level is appropriately set. One interesting point of observation taken from this experiment concerns the emergence of the so-called "fluid compositionality" that supports two seemingly incompatible functions, (i) the algebraic operations on primitives as concrete objects, and (ii) the fluid and context sensitive concatenations of primitives as delicate perceptual spatial-temporal patterns. The very fine structures accommodating these two can be developed in the course of interactive learning between the higher-order cognitive level and the lower perceptual level, where the top-down intentional pathway attempts to generate the whole behavioral flow proactively, and where the bottom-up process modulates such flow by reflecting the perceptual reality. Such interaction

should lead to the achievement of "kinetic melody" [38] in generating fluid, yet compositional actions.

One interesting finding in the fourth experiment was that the pseudo-stochastic process in action-primitive transitions was realized by deterministic chaos self-organized in the higher-level network through the extraction of the probabilistic structure latent in observed visuo-proprioceptive sequences in the lower-level network. What we saw, here, can be understood as a reverse of the ordinary way of constructing the symbolic dynamic [44]-[46]. The symbolic dynamic can be obtained by applying finite numbers of partitions to trajectories of chaos defined in continuous space by assigning a symbol label to each partition. It is known that the symbol sequences obtained by this operation exhibit probabilistic state transition properties which can be reconstructed by stochastic finite state machines with epsilon approximation [46]. The current robotics experiments showed that chaos is self-organized in the adopted neural dynamic system through the process of imitating the observed sequences of primitive actions, sequences that are like stochastic transitions of symbols. Here, the reader may consider that the same phenomena of spontaneous generation of actions, or perceptual sequence patterns, can be generated much more simply by employing probabilistic models such as the Hidden Markov Model [47] or stochastic dynamics models such as in [24]. The crucial difference, however, is that in the case of chaos self-organized in the MTRNN model, the network itself selects each next action deterministically, even though external observers may not be able judge whether it is deterministically or probabilistically determined simply by observing the action sequences generated. On the other hand, in the case of probabilistic or stochastic dynamics models, next actions are selected only probabilistically by means of externally provided random processes. Presumably, deterministic dynamic systems would be indispensable for generating both spontaneous behaviors and intentional behaviors under the same dynamic mechanism, especially by utilizing the initial sensitivity characteristics. Behavioral trajectories departing from arbitrarily set initial states of the intentional units exhibit spontaneous transitions of primitive actions by means of chaos which has been self-organized by extracting the statistical structures from observation. On the other hand, by the deterministic nature of the same model, a particular sequence of shorter length can be generated with rigorous intention by resetting the model to the corresponding initial state values, if saved in the memory. It is also crucial to note that deterministic dynamical systems allow for mean field approximations to neuronal dynamics. Although individual neuronal dynamics may be stochastic, Fokker-Planck formulations and related mean field treatments render the essential dynamics deterministic, again. Accordingly, these deterministic treatments are predominant in the theoretical and modeling literature.

The current research has been inspired by prior studies by others on the dynamical systems approach to understanding cognitive brain processes. First, the current study has been largely influenced by thought about embodied cognition viewed from the dynamical systems perspectives [48]-[50]. It presumes that the essence of embodied cognition should appear in the structural coupling between brains, bodies and environments. Although on this account the perceptual entrainment of internal processes is considered to be the key mechanism ensuring *situatedness* in the environment [50], the current study assumes that sophisticated mechanisms of interaction between top-down prediction and bottom-up error regression are crucial to the development of compositionality in embodied higher-level cognitive processes that also succeed in remaining situated in the outer environment. Recently, other groups have introduced different neuro-dynamic models to deal with compositionality problems in language and behavior association learning [51], and in compositions of sensory-motor primitives into goal-directed actions [52]. However, that research does not focus on the essentiality of top-down and the bottom-up interactions in the development of compositionality. Other brain modeling researchers [53], [54] who regard the brains as complex systems have speculated that chaos might play an important role in cortical processing. For example, some have speculated that memory search processes utilize chaotically generated itinerant trajectories in shifting from one candidate memory state to another, an idea which has some experimental support in EEG recordings of the olfactory bulbs of rabbits smelling odors [55]. Although the observation of spontaneous shifts in generating primitive actions or their motor imagery shown in our robotics experiments might be analogous to this phenomena of so-called "chaos itinerancy" [54], there are fundamental differences in the way that our studies have treated chaos. In our studies, chaos was self-organized by means of reflecting stochasticity observed in primitive action sequences during the learning process. On the other hand, in those brain modeling studies [53], [54], chaos was utilized as additive noise for generating fluctuations in memory dynamics.

Now let us examine possible correspondences of our proposed model, especially the MTRNN, to real brains. The major assumptions represented in the MTRNN are (i) timescale differences in neural activation dynamics are assumed between sub-networks, (ii) a hub-like sub-network is assumed in the intermediate level which is connected both from the higher-level network and the input-level modular networks for motor and vision, and (iii) top-down intention is initiated from the higher-level network and bottom-up error-based recognition originates from the lower-level network. In electrophysiological experiments with monkeys, Tanji and Shima [56] observed timescale differences in the buildup of neural activations between the supplementary motor area (with slower dynamics spanning timescales on the order of seconds) and M1 (with faster dynamics on the order of a fraction of a second) immediately before action generation. Hoshi et al. [57] showed that the time profile of prefrontal cortex neurons during the build-up period can show similar slow dynamics profiles with the ones in the supplementary motor area. Soon et al. [58] demonstrated that brain activity is initiated in the prefrontal cortex up to seven seconds before a conscious decision of free actions in human fMRI imaging experiments. Kiebel et al. [59], Badre and D'Esposito [59], and Uddén and Bahlmann [61] proposed a similar idea to explain the rostral-caudal gradient of timescale differences by assuming slower dynamics at the rostral side (PFC) and faster dynamics at the caudal side (M1) in the frontal cortex to account for a possible functional hierarchy in the region. Next, let us take a glance at the connectivity among the related local cortical areas. First, it is well known that the parietal cortex receives multi-modal sensory information including that of dorsal stream vision (spatial and motion components in the visual inputs) and somatosensation for their possible integration [62]. Also, anatomical studies of monkey brains have shown that the prefrontal cortex and the parietal cortex are densely interconnected [63], [64]. Human brain imaging studies have suggested that the so-called "prefrontal-parietal" network may be involved in executive tasks [65], [66] and attention systems [67]. Taken together, this research suggests that the parietal cortex may serve in the generation of cognitive behaviors by acting as an information hub both for the executive control from the prefrontal cortex and multi-modal sensation from the sensory peripherals.

Integrating present experimental evidence with views prevalent in neuroscience, the following interpretation of the MTRNN and its correspondence with real brains presents itself. Top-down intention for generating voluntary action might be initiated in a slow dynamics network corresponding to the prefrontal cortex. Then, top-down intention might propagate to an intermediate dynamics network corresponding to the parietal cortex, where prediction of the integrated visuo-proprioceptive states expected to result from intended actions is made. This top-down prediction further propagates in one way to the motor cortex where motor commands necessary to achieve the predicted proprioceptive changes (the predicted movements of the limbs) are inversely computed, and in another way to the V5 where visually perceived motion (e.g. movements of a visual object in our robot experiments) is predicted. When the predictions of the

perceptual inputs in vision and proprioception are compared with the actual ones, the resultant errors might be back-propagated to the parietal cortex. This accords with a speculation made by Desmurget and colleagues [68] that the parietal cortex might mediate error monitoring between the predicted perceptual outcome for the intended action and the actual perceptual outcome. Furthermore, they argued that this error monitoring in the parietal cortex may trigger the *postdictive* conscious awareness of the will for generating voluntary actions. Friston [24] provides a similar account, arguing that prediction error causes surprise. I have speculated that our consciousness may arise when the intention in the higher-order cognitive brain is effortfully modified through the process of searching for the error minimum [69]-[71].

Future research should concern the scaling issues in the models presented in the current paper. The experiment on associative learning between language and action should include more complex sentences, such as those composed of an objective noun followed by adverb phrase like "Put A on B". It is assumed that generalized learning of such a case should require an additional principal dimension in the concept space (a 3-dimensional rather than a 2-dimensional grid) generated in the PB mapping. An interesting question might be how much the concept space can be enlarged just by adding further principal dimensions to the space. Other possible scenarios should be considered, also, because any incremental increase in the number of essential dimensions would cause a combinatorial explosion where generalization of learning might become too difficult to be achieved. For the MTRNN model, scalability issues such as the incremental increases in memory size in terms of the number of different primitives to be acquired versus the incremental increase in the number of neural units in the network are important, but there are other fundamental issues open to future research as well. One concerns the setting of the time constant parameters. Although, in the current study, the time constant parameter of each level was intuitively set by the experimenters, it is more desirable that the parameter can be set automatically. Although it is mathematically possible to optimize each time constant by means of the error minimization, it is not certain how much more stably such optimization processes can converge. The clarification of this problem, alone, requires some intensive studies. Another issue concerns the number of levels to be determined for particular given tasks. Although there might be some optimization methods to determine the number by learning, an open question is if the levels should be maintained as countable ones, or if the functionality should change rather continuously without having explicit boundaries between them. It should be also true that the degree of topological connectivity between different levels affects the degree of segregation between them [72]. Dense connectivity

will suppress developments of independent functions at each level, whereas sparse connectivity will suppress necessary interactions between them. Theoretically, it may be possible to optimize the connectivity, the number of hierarchical levels, and the number of nodes using Bayesian model comparison. This follows because our scheme is formally related to free energy minimization in active inference [24]. This is important because the free energy provides an objective function that can be used not just to make perceptual inferences and to optimize connection weights, but it can also be used to score different models of the embodied environment - and therefore to optimize the model itself. Although this has not been demonstrated in practice, it remains an exciting prospect.

### **APPENDIX-A**

## Maximum Lyapunov Exponent

The maximum Lyapunov exponent of a dynamic system represents the rate of exponential divergence from the perturbed initial conditions. By considering two points,  $X_0$  and  $X_0 + \delta X_0$ , in a state space, each of which develops in time to generate an orbit, the maximum Lyapunov exponent  $\lambda$  can be defined as

$$\lambda = \lim_{t \to \infty} \frac{1}{t} \ln \frac{\delta X_t}{|\delta X_0|},$$

where  $\delta X_0$  represents the initial separation vector of two orbits, and  $\delta X_t$  is the separation vector at time *t*.

For evaluating the maximum Lyapunov exponent for each sub-network, 100 sample sequences of 100,000 time steps with random initial states and initial separation vectors were generated by performing the closed-loop operation of the whole network without receiving the external inputs. When the maximum Lyapunov exponent of the lower- or the higher-level network was measured, we computed the dynamics of the entire network, but evaluated a separation vector containing only the component of a subnetwork as the lower- or the higher-level component. This method measures the contribution of the subnetwork to the initial sensitivity of the dynamics. Note that if the subnetwork has a positive Lyapunov exponent, as measured in the abovementioned manner, then the entire network also has a positive Lyapunov exponent.

## Acknowledgement

The author would like to thank Prof. Karl Friston for providing us valuable advice on the manuscript. He thanks also to Dr. Jeffrey White for carefully examining the English expressions in the manuscript. This work was supported by the KAIST Settlement and Research of New Instructors fund for the Year 2012, with the research title "Nero-Robotics Experiments with Large Scale Brain Networks".

#### References

[1] G. Evans, The Varieties of Reference. Oxford: Clarendon Press. 1982.

M. A. Arbib, "Perceptual structures and distributed motor control," *Handbook of Physiology : The Nervous System, vol. II*,. Cambridge, MA: MIT Press, pp.1448-1480, 1981.

[3] S. Harnad, "The symbol grounding problem," *Physica D: Nonlinear Phenomena*, vol. 42, no. 1, pp.335-346, 1990.

[4] Z. W. Pylyshyn, *The robot's dilemma: The frame problem in artificial intelligence.* Norwood NJ: Ablex, 1987.

[5] A. Cangelosi, A. Greco and S. Harnad, "From robotic toil to symbolic theft: grounding transfer from entry-level to higher-level categories," *Connection Science*, vol.12, no. 2, pp. 143-164, 2000.

[6] A. Miyake, N.P. Friedman, M. J. Emerson, A. H. Witzki, A. Howerter and T. D. Wager, "The unity and diversity of executive functions and their contributions to complex "frontal lobe" tasks: A latent variable analysis," *Cognitive psychology*, vol. 41 no. 1, pp. 49-100, 2000.

[7] E. Koechlin, C. Ody and F. Kouneiher, "The architecture of cognitive control in the human prefrontal cortex," *Science*, vol.302, no. 5648, pp. 1181-1185, 2003.

[8] Joaquin M. Fuster, *The prefrontal cortex*, Academic Press, 2008.

[9] D. J. Felleman, D. C. Van Essen, "Distributed hierarchical processing in the primate cerebral cortex," *Cerebral cortex*, vol. 1, no. 1 pp. 1-47, 1991.

[10] K. Tanaka, "Neuronal mechanisms of object recognition," *Science*, vol. 262, no. 5134, pp. 685-688, 1993.

[11] J. M. Fuster, "Prefrontal neurons in networks of executive memory," *Brain research bulletin*, vol. 52, no.5, pp.331-336, 2000.

[12] H. Liepmann, "Apraxie", Erg ges Med, vol. 1, pp. 516–543, 1920.

[13] K. M. Heilman, L. J. Rothi and E. Valenstein, "Two forms of ideomotor apraxia," *Neurology*, vol. 32, no. 4, pp. 342, 1982.

[14] A. F. D. C. Hamilton and S. T. Grafton, "Action outcomes are represented in human inferior frontoparietal cortex," *Cerebral Cortex*, vol. 18, no.5, pp. 1160-1168, 2008.

[15] H. B. Barlow, "Single units and sensation: a neuron doctrine for perceptual psychology," *Perception*, vol. 1, pp. 371-394, 1972.

[16] C. G. Gross, "Genealogy of the "grandmother cell"," The Neuroscientist, vol. 8 no. 5,

pp. 512-518, 2002.

[17] J. Tani and M. Ito, "Self-organization of behavioral primitives as multiple attractor dynamics: a robot experiment," *IEEE Trans. on Sys. Man and Cybern. Part A*, vol. 33, no. 4, pp. 481–488, 2003.

[18] J. Tani, "Learning to generate articulated behavior through the bottom-up and the top-down interaction process," *Neural Networks*, vol. 16, pp. 11–23, 2003.

[19] J. Tani, M. Ito, and Y. Sugita, "Self-organization of distributedly represented multiple behavior schema in a mirror system," *Neural Netw.*, vol. 17, pp. 1273–1289, 2004

[20] J.L. Elman, "Finding structure in time. Cognitive Science," Vol. 14, pp. 179–211, 1990.

[21] M.I. Jordan, "Attractor dynamics and parallelism in a connectionist sequential machine,"

In Proc. of Eighth Annual Conference of Cognitive Science Society, pp. 531-

546. Hillsdale, NJ: Erlbaum, 1986.

[22] D.E. Rumelhart, G.E. Hinton, and R.J. Williams, "Learning internal representations by error propagation," In D.E. Rumelhart and J.L. Mclelland, editors, *Parallel Distributed Processing*. Cambridge, MA: MIT Press, 1986.

[23] R. P. N. Rao and D. H. Ballard, "Predictive coding in the visual cortex: a functional interpretation of some extra-classical receptive-field effects," *Nature Neuroscience*, vol. 2, pp. 79-87, 1999

[24] K. Friston, "The free-energy principle: a unified brain theory?," *Nature Reviews Neuroscience*, vol. 11, pp.127-138, 2010.

[25] R. J. Williams and D. Zipser, "A Learning algorithm for continually running fully recurrent neural networks," *Neural Computation*, vol. 1, pp. 270-280, 1989.

[26] K. Doya and S. Yoshizawa, "Memorizing oscillatory patterns in the analog neuron network," *Proc. of the 1989 International Joint Conference on Neural Networks*, vol. I, pp.27-32, *IEEE Press*, 1989.

[27] Y. Yamashita and J. Tani, "Emergence of functional hierarchy in a multiple timescale neural network model: a humanoid robot experiment," *PLoS Computational Biology*, vol. 4, no.11, 2008.

[28] R. Nishimoto, J. Tani, ""Development of hierarchical structures for actions and motor imagery: a constructivist view from synthetic neurorobotics study," Psychological Research, vol. 73, pp. 545-558, 2009.

[29] J. Namikawa, R. Nishimoto and J. Tani, "A neurodynamic account of spontaneous behaviour," *PLoS Computational Biology*, vol. 7, no. 10, e1002221, 2011.

[30] M. Ito, K. Noda, Y. Hoshino and J. Tani, "Dynamic and interactive generation of object handling behaviors by a small humanoid robot using a dynamic neural network model," *Neural Networks*, vol. 19, no. 3, pp. 323-337, 2006.

[31] O. Hauk, I. Johnsrude and F. Pulvermuller, "Somatotopic representation of action words in human motor and premotor cortex," *Neuron*, vol.41, no. 2, pp.301–307. 2004.

[32] M. Tettamanti, G. Buccino, M.C. Saccuman, V. Gallese, M. Danna, P. Scifo, F. Fazio, G. Rizzolatti, S.F. Cappa and D. Perani, "Listening to action-related sentences activates fronto-parietal motor circuits," *Journal of Cognitive Neuroscience*, vol.17, no. 2, pp.273-281, 2005.

[33] F. Pulvermuller, "Brain mechanisms linking language and action. Nature Reviews," *Neuroscience*, vol. 6, pp. 576–582, 2005.

[34] M. Tomasello, *Constructing a language: A usage-based theory of language acquisition*. Harvard University Press, Cambridge, MA, 2003.

[35] J. A. Fodor and Z. W. Pylyshyn, "Connectionism and cognitive architecture: A critique," *Cognition*, vo. 28, pp.3-71, 1988.

[36] Y. Sugita and J. Tani, "Learning semantics combinatoriality from the interaction between linguistic and behavioral processes," *Adaptive Behavior*, vol. 13, no. 1, pp. 33-52, 2005.

[37] J. Tani. (2014, April). Robot movies are available in the online supplementary material at http://neurorobot.kaist.ac.kr/video\_ieee2014.html.

[38] A.R. Luria, The Working Brain, Penguin Books Ltd., 1973.

[39] T. Kohonen, "Self-organized formation of topologically correct feature maps," *Biological cybernetics*, vol. 43, no. 1, pp.59-69, 1982.

[40] J. R. Saffran, R. N. Aslin and E. L. Newport, "Statistical learning by 8-month-old infants," *Science*, vol.274, pp.1926-1928, 1996.

[41] N. Z. Kirkham, J. A. Slemmer and S. P. Johnson, "Visual statistical learning in infancy: Evidence for a domain general learning mechanism," *Cognition*, 83, pp. 35-42, 2002.

[42] D. Baldwin, A. Andersson, J. Saffran and M. Meyer, "Segmenting dynamic human action via statistical structure," *Cognition*, vol. 106, pp. 1382-1407, 2008.

[43] W. J. Freeman, *How brains make up their minds?* New York, Columbia University Press, 2000.

[44] M. Morse and G. A. Hedlund, "Symbolic dynamics," *American Journal of Mathematics*, vol. 60, no. 4, pp.815-866, 1938.

[45] S. Wiggins, *Global Bifurcations and Chaos Analytical Methods* (Applied Mathematical Sciences, vol. 73), Springer-Verlag, New York, 1988.

[46] J. P. Crutchfield, and K. Young, "Inferring statistical complexity," *Physical Review Letters*, vol. 63, no. 2, pp. 105-108, 1989.

[47] L. E. Baum and T. Petrie, "Statistical Inference for Probabilistic Functions of Finite State Markov Chains," *The Annals of Mathematical Statistics*, vol. 37, no. 6, pp. 1554—1563, 1966.

[48] G. Schoner and J. A. S. Kelso, "Dynamic pattern generation in behavioral and neural systems," *Science*, vol. 239, pp. 1513-1539, 1988.

[49] E. Thelen and L. Smith, A dynamic systems approach to the development of cognition and action, MIT Press, 1994.

[50] R. D. Beer, "Dynamical approaches to cognitive science," *Trends in Cognitive Sciences*, vol. 4, no. 3, pp. 91-99, 2000.

[51] E. Tuci, T. Ferrauto, A. Zeschel, G. Massera and S. Nolfi, "An Experiment on behaviour generalisation and the emergence of linguistic compositionality in evolving robots," *IEEE Transactions on Autonomous Mental Development*, vol. 3, no. 2, pp. 176-189, 2011.

[52] Y. Sandamirskaya, S. K. Zibner, S. Schneegans and G. Schöner, "Using dynamic field theory to extend the embodiment stance toward higher cognition," *New Ideas in Psychology*, vol. 31, no. 3, pp. 322-339, 2013.

[53] K. Aihara, "Chaotic neural networks," *Physical Letters A*, vol. 144, no. 6, pp. 333–340, 1990.

[54] I. Tsuda, E. Koerner, and H. Shimizu, "Memory dynamics in asynchronous neural networks," *Prog. Theor. Phys.*, vol. 78, pp. 51–71, 1987.

[55] C. A. Skarada and W. J. Freeman, "Does the brain make chaos in order to make sense of the world?," *Behavioral and Brain Sciences*, vol. 10, pp. 161–165, 1987.

[56] J. Tanji and K. Shima, "Role for supplementary motor area cells in planning several movements ahead," *Nature*, vol. 371, pp. 413-416, 1994.

[57] E. Hoshi, K. Shima and J. Tanji, "Neuronal activity in the primate prefrontal cortex in the process of motor selection based on two behavioral rules." *Journal of Neurophysiology*, vol. 83, no. 4, pp.2355-2373, 2000.

[58] C. Soon, M. Brass, H. Heinze and J. Haynes, "Unconscious determinants of free decisions in the human brain", *Nature Neuroscience*, vol. 11, pp.543-545, 2008.

[59] S. J. Kiebel, J. Daunizeau and K. J. Friston, "A hierarchy of time-scales and the brain." *PLoS Computational Biology*, vol. 4, e1000209, 2008.

[60] D. Badre and M. D'Esposito, "Is the rostro-caudal axis of the frontal lobe hierarchical?" *Nature Reviews Neuroscience*, vol. 10, pp. 659-669, 2009.

[61] J. Uddén and J. Bahlmann, "A rostro-caudal gradient of structured sequence

processing in the left inferior frontal gyrus," *Philos. Trans. R. Soc. Lond. B Biol. Sci.*, vol. 367, pp. 2023–2032, 2012.

[62] S. J. Blakemore and U. Frith, *The Learning Brain*. Blackwell Publishing, 2005.

[63] L. D. Selemon and P. S. Goldman-Rakic, "Common cortical and subcortical targets of the dorsolateral prefrontal and posterior parietal cortices in the rhesus monkey: evidence for a distributed neural network subserving spatially guided behavior," *The Journal of Neuroscience*, vol. 8, no. 11, pp. 4049-4068, 1988.

[64] P. S. Goldman-Rakic, "Topography of cognition: parallel distributed networks in primate association cortex," *Annual review of neuroscience*, vol. 11, no. 1, pp.137-156, 1988.

[65] R. Cabeza and L. Nyberg, "Imaging cognition II: an empirical review of 275 PET and fMRI studies." *Journal of Cogn. Neurosci.*, vol. 12, no. 1, pp. 1-47, 2000.

[66] J. Duncan and A. M. Owen, "Common regions of the human frontal lobe recruited by diverse cognitive demands," *Trends Neurosci.*, vol. 23, no. 10, pp. 475-483, 2000.

[67] M. I. Posner, "Attention: The mechanisms of consciousness.", *Proceedings of the National Academy of Sciences USA*, vol. 91, pp.7398-7403, 1994.

[68] M. Desmurget, K.T. Reilly, N. Richard, A. Szathmari, C. Mottolese and A. Sirigu,
"Movement intention after parietal cortex stimulation in humans," *Science*, vol. 324, pp. 811-813, 2009.

[69] J. Tani, "An interpretation of the `Self' from the dynamical systems perspective: A constructivist approach", *Journal of Consciousness Studies*, Vol.5, No.5/6, pp.516-542, 1998.

[70] J. Tani, "The dynamical systems accounts for phenomenology of immanent time: An interpretation by revisiting a robotics synthetic study", *Journal of Consciousness Studies*, vol.11, no.9, pp.5-24, 2004.

[71] J. Tani, "On the interactions between top-down anticipation and bottom-up regression," *Frontiers in neurorobotics*, vol. 1, 2007.

[72] R. W. Paine and J. Tani, "How hierarchical control self-organizes in artificial adaptive systems," *Adaptive Behavior*, vol. 13, no. 3, pp. 211-225, 2005.