Emergence of Interactive Behaviors between Two Robots by Prediction Error Minimization Mechanism

Yiwen Chen, Shingo Murata, Hiroaki Arie,
Tetsuya Ogata, Jun Tani, and Shigeki Sugano *

Abstract

This study demonstrates that the prediction error minimization (PEM) mechanism can account for the emergence of reciprocal interaction between two cognitive agents. During interactive processes, alternation of forming and deforming interactions may be triggered by various internal and external causes. We focus in particular on external causes derived from a dynamic and uncertain environment. Two small humanoid robots controlled by an identical dynamic neural network model using the PEM mechanism were trained to achieve a set of coherent ball-playing interactions between them. The two robots predict each other in a top-down way while they try to minimize the prediction errors derived from the unstable ball dynamics or the external cause in a bottom-up way by using the PEM mechanism. The experimental results showed that switching among the set of trained interactive ball plays between the two robots appears spontaneously. The analysis clarified how each complementary behavior can be generated via mutual adaptation between the two robots by undertaking top-down and bottom-up interaction in each individual dynamic neural network model by using the PEM mechanism.

1 Introduction

Humans are interdependent agents that interact with others. As an example of human interactions, consider a situation where two children are rolling a ball between themselves. Once organized, the cooperative ball-playing interaction might be compulsorily deformed due to two different kinds of causes. One kind is an internal cause, such as one child deciding to monopolize the ball. The other kind is an external cause generated by the environment, such as the ball unpredictably rolling beyond the children's control. Due to the many possible causes, the alternation of forming and deforming interaction can appear in a spontaneous way.

Several studies have conducted experiments on interactions between two agents in simulated environments [1–3] and between robots in physical environments [4, 5]. For example, Ikegami and Iizuka [1] demonstrated the emergence of turn-taking behavior between two agents, referred to as coupled dynamical recognizers [6], each of which is equipped with a single recurrent neural network (RNN) [7, 8]. This computer simulation dealt with simple turn-taking behavior between a leader and a follower in a two-dimensional space. Hinoshita et al. [4] used RNNs to

realize multi-modal interactions between two robots with voice and motion. In this framework, each robot is equipped with two associated RNNs, one for voice and the other for motion generation. Although this demonstrated more complex interactions between robots in a physical environment, turn-taking between a speaker/actor and a listner/observer must be explicitly performed by an experimenter. In this study, we speculate that an additional mechanism, namely prediction error minimization (PEM) [9–12], is essential to understanding the emergent aspect of interactions between two cognitive agents in a physical environment.

PEM can be implemented by a computational framework called predictive coding [13] or predictive processing [14], which performs dense interactions between top-down predictive and bottom-up recognition processes. In the field of theoretical neurobiology, Friston et al. [15] proposed a Bayesian framework called active inference in which both action and perception aim to minimize prediction errors by changing sensory inputs and predictions, respectively. Based on this framework, Friston and Frith [16, 17] simulated birdsong communication between synthetic songbirds or two agents with the PEM mechanism. In the field of cognitive robotics, Tani [9] proposed a connectionist framework called RNN with parametric bias (RNNPB), where PB is a static vector attached to the conventional RNN. On the basis of this framework, Noda et al. [18] demonstrated flexible switching of object handling behaviors by a humanoid robot with the PEM mechanism. In their work, the robot first learned two ball-playing behaviors, "rolling a ball" and "lifting a ball," depending on the ball dynamics by optimizing the respective PB. After the learning process, the robot could switch its behavior by inferring the PB that minimizes prediction errors generated by unstable ball dynamics or external causes. Although these studies clarified how the PEM mechanism can work effectively in some cognitive tasks, the former showed that an agreement between two agents can be reached by the PEM in a simple simulation setting and the latter showed that a complex but a single humanoid robot can achieve coherent interaction with the environment (a ball) in the physical setting.

The current study considers the interaction between two cognitive agents with the PEM mechanism situated in a physical environment. The beginning of this section describes the internal and external causes for forming and deforming interactions. For simplicity, we focus on the influence of external causes from the surrounding environment. For this purpose, we extend the experiment on the switching of ball-playing behaviors by Noda et al. [18], which also considered the influence of external causes derived from unstable ball dynamics. We employ two robots, each implemented with an RNN-based model with the PEM mechanism in the environment. Each robot first learned a set of ball-playing behaviors through interaction with a human experimenter, and then encountered each other. In the first interaction, the human experimenter provides an external cause to deform the current interactive behavior. The robot is evaluated with and without the PEM mechanism to determine whether it can form the corresponding interactive behavior. This forcibly deformed interaction is unidirectional, as demonstrated in [19, 20]. The second experiment considers bidirectional interaction between the two robots with the PEM mechanism in the environment. In this experiment the two robots, each of which tries to minimize prediction errors, influence each other. The experimental results demonstrate the emergent and spontaneous aspects of reciprocal interaction in terms of the PEM mechanism.

2 Computational Model

As a connectionist framework to realize the PEM mechanism between two robots, this study adopted a stochastic continuous-time RNN (S-CTRNN) [19, 21], in which we assigned several context units as PB units. S-CTRNNs can learn to generate predictions about the mean and variance of the succeeding sensory states, which are assumed to follow a Gaussian distribution,

by receiving the current states. The variance prediction mechanism enables an S-CTRNN to achieve more stable learning of target data with fluctuations than the conventional CTRNN can, as demonstrated in [22].

The following subsections describe the forward dynamics of each neural unit and optimization method with the PEM mechanism in the learning and generation phases.

2.1 Forward Dynamics

The internal state $u_{t,i}^{(s)}$ $(1 \leq t)$ of each neural unit is described by

$$
u_{t,i}^{(s)} = \begin{cases} u_{t-1,i}^{(s)} & (i \in I_{\mathrm{P}}), \\ \left(1 - \dfrac{1}{\tau_i}\right) u_{t-1,i}^{(s)} + \dfrac{1}{\tau_i} \left(\displaystyle\sum_{j \in I_{\mathrm{I}}} w_{ij} x_{t,j}^{(s)} + \sum_{j \in I_{\mathrm{C}}} w_{ij} c_{t-1,j}^{(s)} + \sum_{j \in I_{\mathrm{P}}} w_{ij} p_{t,j}^{(s)} + b_i \right) & (i \in I_{\mathrm{C}}), \\ \displaystyle\sum_{j \in I_{\mathrm{C}}} w_{ij} c_{t,j}^{(s)} + b_i & (i \in I_{\mathrm{O}} \cup I_{\mathrm{V}}). \end{cases}
\tag{1}
$$

Here, I_{I}, I_{P}, I_{C}, I_{O}, and I_{V} are the index sets for the input, PB, context, output, and variance units, τ_i is the time constant of the ith context unit, w_{ij} is the connection from the jth to the ith unit, $x_{t,j}^{(s)}$ is the jth input state at time step t of the sth sequence, $c_{t,j}^{(s)}$ is the jth context state, $p_{t,j}^{(s)}$ is the jth PB state, and b_i is the bias of the ith unit. From this equation, the PB states can be regarded as a particular case of the context states, whose time constant is infinite. In this study, the value of the initial internal state $u_{0,i}^{(s)}$ of the context units $(i \in I_{\mathrm{C}})$ was set to zero, indicating a neutral state independent of the temporal sequence s. In contrast, that of the PB units $(i \in I_{\mathrm{P}})$ was optimized for each sequence in the learning phase, as described later.

The internal state of each unit is activated by using the respective nonlinear functions as follows:

$$
\begin{align}
p_{t,i}^{(s)} &= \tanh(u_{t,i}^{(s)}) & (i \in I_{\mathrm{P}}), \tag{2} \\
c_{t,i}^{(s)} &= \tanh(u_{t,i}^{(s)}) & (i \in I_{\mathrm{C}}), \tag{3} \\
y_{t,i}^{(s)} &= \tanh(u_{t,i}^{(s)}) & (i \in I_{\mathrm{O}}), \tag{4} \\
v_{t,i}^{(s)} &= \exp(u_{t,i}^{(s)}) & (i \in I_{\mathrm{V}}). \tag{5}
\end{align}
$$

2.2 Optimization Method

Under the Gaussian assumption, we can write the following objective function of the negative log-likelihood by using the target $\hat{y}_{t,i}^{(s)}$, output (mean) $y_{t,i}^{(s)}$, and variance $v_{t,i}^{(s)}$ states (up to constant terms):

$$
\mathcal{L}_{t,i}^{(s)} = \frac{\ln v_{t,i}^{(s)}}{2} + \frac{(\hat{y}_{t,i}^{(s)} - y_{t,i}^{(s)})^2}{2 v_{t,i}^{(s)}}.
\tag{6}
$$

This negative log-likelihood is formally equivalent to the free energy employed in the active inference scheme [15]. From the equation, we can see that minimizing the objective function corresponds to minimizing the variance or uncertainty and (precision-weighted) prediction error. In what follows, we consider two ways of minimizing the function by accumulating it over the long term in the learning phase and over the short term in the generation phase.

In both the phases, parameters or PB states at epoch n collected by $\boldsymbol{\theta}_n$ are updated using a gradient descent on an accumulated negative log-likelihood $\mathcal{L}$:

$$\boldsymbol{\theta}_n = \boldsymbol{\theta}_{n-1} + \alpha \Delta \boldsymbol{\theta}_n, \tag{7}$$

$$\Delta \boldsymbol{\theta}_n = -\frac{\partial \mathcal{L}}{\partial \boldsymbol{\theta}} + \eta \Delta \boldsymbol{\theta}_{n-1}. \tag{8}$$

Here, α is the learning rate and η is a parameter representing the momentum term. The negative log-likelihood accumulated in a different way in each phase is described below.

2.2.1 Learning Phase

In the learning phase, all time-invariant parameters, including connections w_{ij}, biases b_i, and initial internal states of the PB units $u_{0,i}^{(s)}$ $(i \in I_\mathrm{P})$, which are collected by $\boldsymbol{\theta}$ above, are optimized off-line using recorded target sequences. The optimization is performed by minimizing the following sum of $\mathcal{L}_{t,i}^{(s)}$ with respect to all the dimensions, time steps, and sequences:

$$\mathcal{L} = \sum_{s \in I_\mathrm{S}} \sum_{t=1}^{T^{(s)}} \sum_{i \in I_\mathrm{O}} \mathcal{L}_{t,i}^{(s)}. \tag{9}$$

Here, I_S is the data index set, and $T^{(s)}$ is the length of the sth temporal sequence. The gradient of the objective function with respect to each parameter can be obtained by back-propagation through time (BPTT) [23], as described in [21].

2.2.2 Generation Phase

In the generation phase after the learning phase, only the internal states of the PB units at time step $t-W$ $(u_{t-W,i}^{(s)})$ are allowed to be optimized on-line, and the other parameters are fixed. The optimization is performed by minimizing the following sum of $\mathcal{L}_{t',i}^{(s)}$ over the immediate past:

$$\mathcal{L} = \sum_{t'=t-W+1}^{t} \sum_{i \in I_\mathrm{O}} \mathcal{L}_{t',i}^{(s)}. \tag{10}$$

Here, W is the length of the time window moving along the increment of the time step t'.

3 Neurorobotics Experiment

3.1 Task Design

We designed a ball-playing interaction between two small humanoid robots (NAO; Aldebaran Robotics). Figure 1 shows a schematic illustration of our framework for a neurorobotics experiment in which both the robots (Robot 1 and Robot 2) were simultaneously controlled by an identical S-CTRNN. These robots with the PEM mechanism first used the S-CTRNN to learn a set of ball-playing behaviors off-line by using recorded data obtained in interaction between the robot and a human experimenter. In this learning phase, the S-CTRNN learned the relationship between visual and proprioceptive states by optimizing connections, biases, and PB states. After predictive learning of visuo-proprioceptive states, the human experimenter was replaced with the other robot and the two robots interacted. Unstable ball dynamics in the real environment provided an external cause triggering switching of PB states for minimizing prediction errors to adapt to the current situation. In other words, the ball dynamics stimulated the unpredictable alternation of forming and deforming interactive behaviors and led to spontaneous interaction.
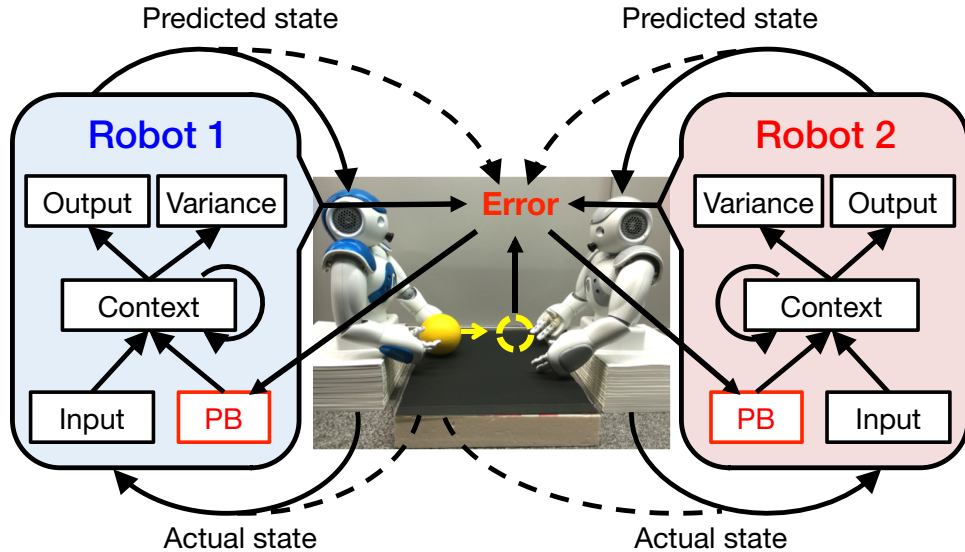
4

Figure 1. Framework for a neurorobotics experiment. The robot on the left is Robot 1, and that on the right is Robot 2. The solid lines of the actual and predicted states represent proprioception and the dotted lines represent vision.

3.2 Interactive Behavioral Patterns

We considered four behavioral patterns, shown in Fig. 2. These patterns can be classified into two categories according to coordination level. The first class, whose conformity is high, consists of rolling the ball with the right (R) and the left (L) hand. This class is characterized in that the timing of these behaviors strongly depends on the partner. Therefore, the robot must learn the relation between self and environment and wait for the ball to come. The other class involves self-play (S) and attraction (A), whose conformity is low. These behaviors can be freely executed, because there is no conflict between them. These behavioral patterns were represented by 10-dimensional time-series data consisting of two-dimensional visual states (the ball position in a visual image) and eight-dimensional proprioceptive states (four for each of the left and right arm).

3.3 Parameter Setting for the Experiment

The numbers of the input, output, and variance units of the S-CTRNN were $N_{\mathrm{I}} = N_{\mathrm{O}} = N_{\mathrm{V}} = 10$. These were determined by the dimension of the robot's visuo-proprioceptive states. The number and time constant of context units were $N_{\mathrm{C}} = 50$ and $\tau_{\mathrm{C}} = 4$, respectively. There were two context units assigned as PB ($N_{\mathrm{P}} = 2$) whose time constant was infinite. In the learning phase, the parameters collected by $\boldsymbol{\theta}$ were optimized off-line for $300,000$ times. In the generation phase, the internal states of the PB units at time step $t - W$ ($u^{(s)}_{t-W,i}$) were optimized on-line for 20 times where the window length was $W = 20$.

4 Results and Discussion

To test the capability of the PEM mechanism, we compared the results from two experiments. In the first case, a human experimenter interacted with a robot trained with S-CTRNN in the physical environment in the same way as in the data-recording phase. The experimenter acted on the environment to provide an external cause (a change of ball dynamics) as a trigger
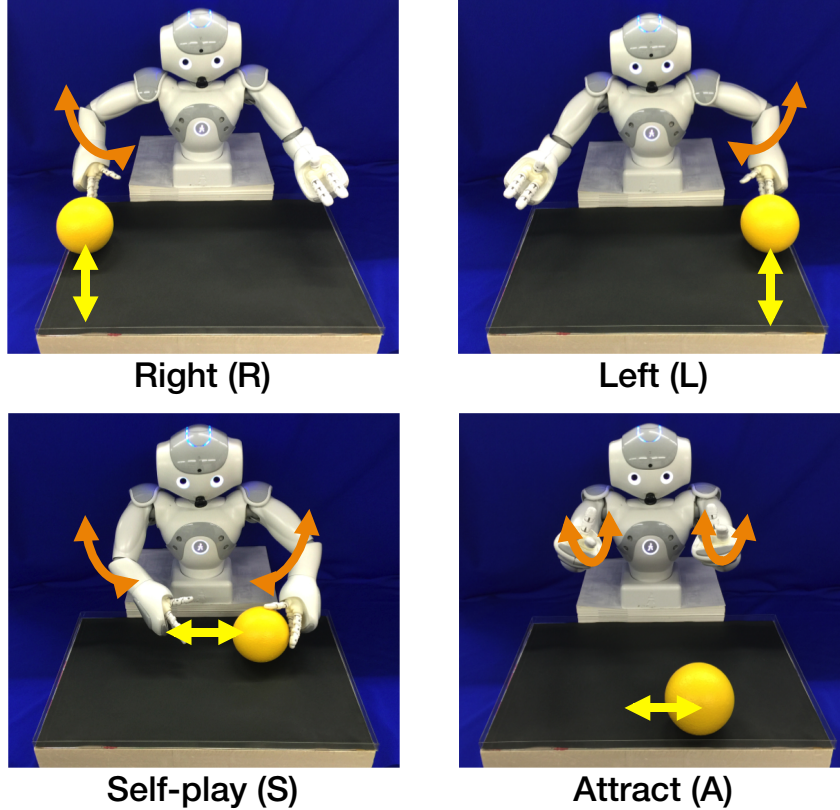
Figure 2. Four interactive behavioral patterns that each robot learned with the S-CTRNN in the experiment. The upper-left and right figures show the ball-rolling behaviors with the right (R) and the left (L) hand, respectively. The lower-left and right figures show the self-play (S) and attract (A) behaviors.

for deforming the ongoing interactive behaviors. We observed whether the robot could switch behavioral patterns in response to the change of the environment and evaluated the generation ability of the robot with and without the PEM mechanism. We expected the results to elucidate the importance of the PEM mechanism in reforming interactive behavior from the deformed status. The second experiment simultaneously involved the two robots with the PEM mechanism. The two robots spontaneously influenced each other to demonstrate phenomena occurring between the robots. We anticipated that the PEM mechanism would initiate reciprocal interactive behavior between the two robots.

4.1 Interaction between the Robot and the Experimenter

Figure 3 shows the generated results of the robot with and without the PEM mechanism. An experimenter interacted with the robot by manually rolling the ball to a place that the robot had previously learned. Within the initial 250 time steps, the experimenter cooperated with the robot to complete the complementary ball-playing interaction. In both cases, the robot could predict the correct values for proprioception and vision with relatively low prediction error. This phenomenon indicates successful forming of interactive behavior between the robot and the experimenter.

However, when the experimenter changed the position of the ball, the robot without the PEM mechanism could not switch behaviors or interact with the experimenter. In the case without the PEM mechanism, the PB dynamics were unable to be optimized, retaining initial values. In contrast, the robot with the PEM mechanism mutually adapted to the experimenter
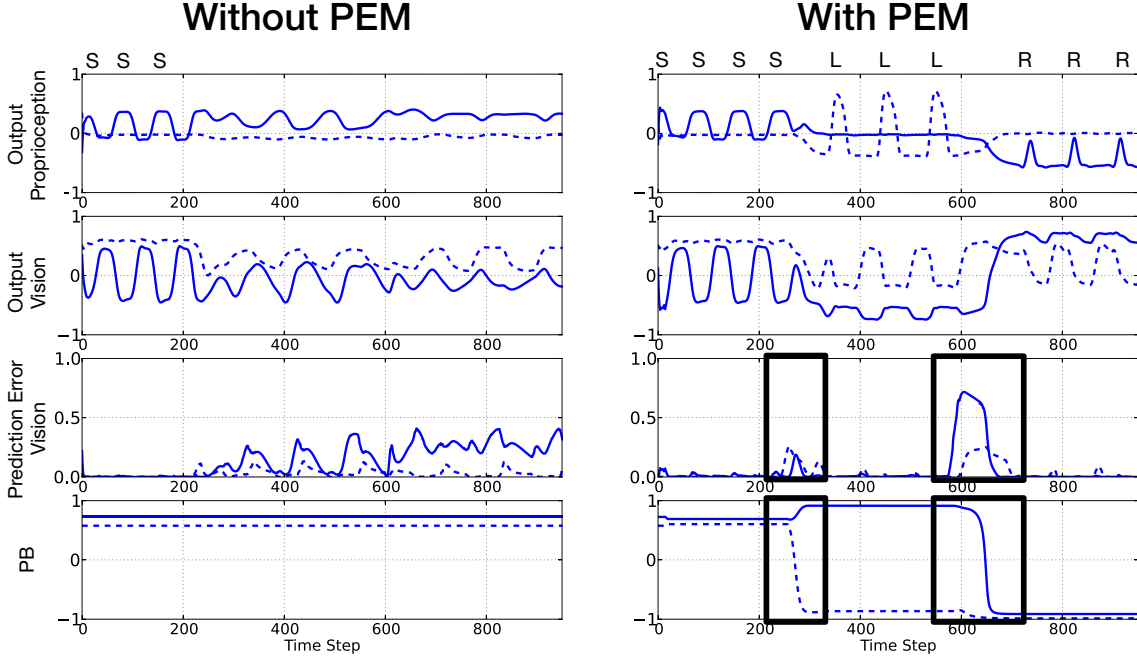
Figure 3. The generated results of robots interacting with the human experimenter. The right and left sides respectively show the case with and without the PEM mechanism. The figures in the first row extract the output results of proprioception from eight dimensions to two dimensions. The figures in the second and third rows respectively show the vision output states and the corresponding prediction error. The figures in the last row show the PB states.

and inferred complementary behaviors from experience. The PB states were dynamically determined in the direction of minimizing the prediction error due to changes in ball position. The PEM mechanism thus initiated interactive behavior between the robot and experimenter. This phenomenon can be regarded as unidirectional interaction in which one agent (the robot) was influenced by a companion agent (the human experimenter).

4.2 Interaction between the Two Robots

After confirming the generation ability of a robot with the PEM mechanism while interacting with a human experimenter, we employed the same framework to the two robots. Figure 4 depicts the generated results of interactive behavior between the two robots. Within the initial 250 time steps, both robots indeed generated behavioral patterns corresponding with the PB state and retained low prediction errors. This period was taken as the realization of forming interaction.

However, when the ball abruptly rolled to an unexpected location owing to a collision between the ball and a robot's hand, both robots' predictions failed. The interaction was deformed and the prediction error became extremely large. To adapt to this perturbation, the PEM mechanism optimized the PB states by following the direction of minimizing the prediction error. Robot 1 achieved a behavior switch from self-play (S) to left (L), and Robot 2 switched its behavior from attract (A) to right (R). In accordance with their past learning experience, both robots successfully switched their behavioral patterns, thereby minimizing the prediction error. The interaction was well organized, and the two robots were able to maintain an interrelated relationship.

At around time step 650, the ball again rolled to an unexpected position from one side of
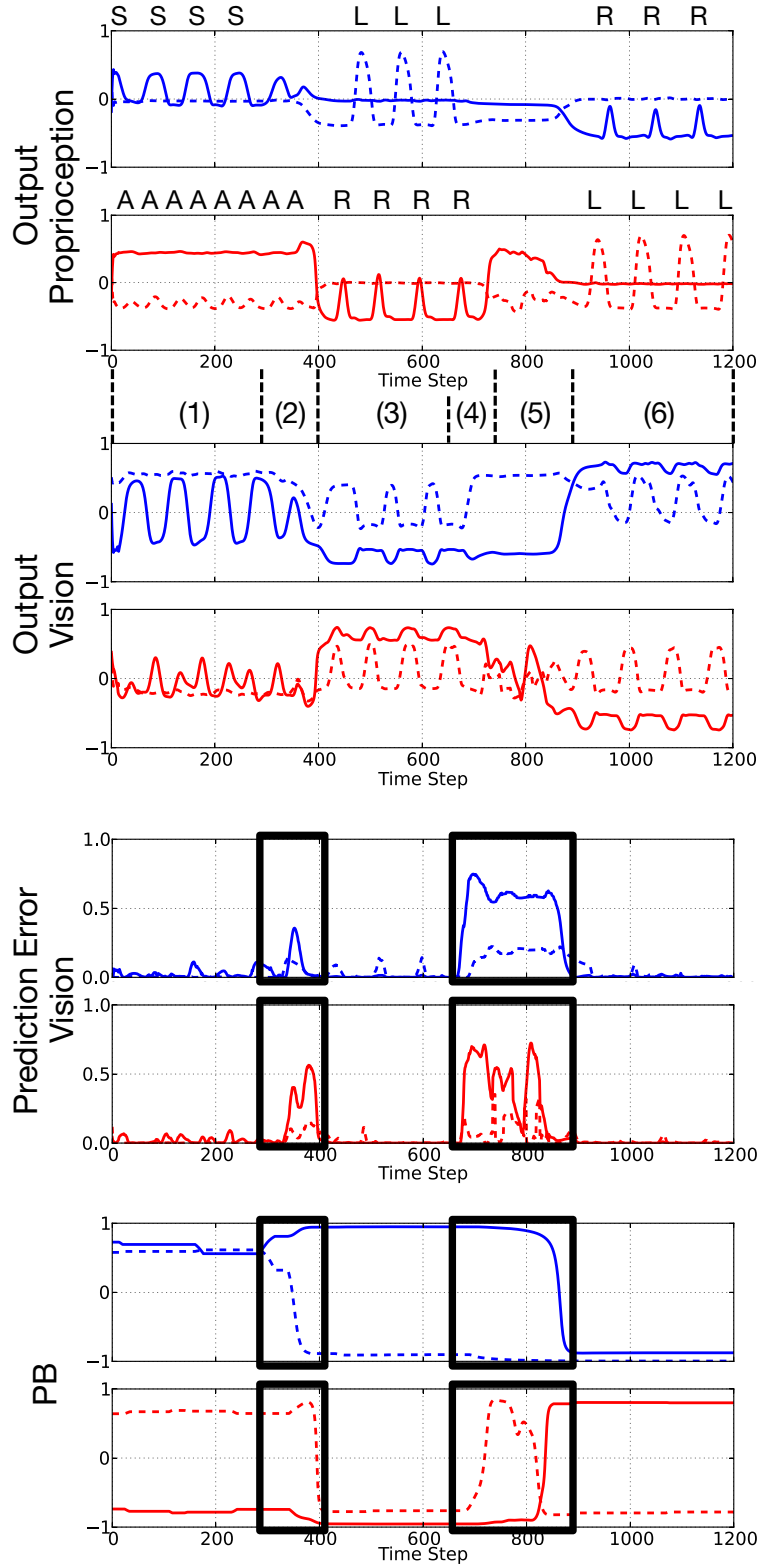
Figure 4. Interaction between Robot 1 and Robot 2, with results separately shown as blue and red lines. The first two panels show the output states for proprioception. For clarity, two of the eight dimensions in the proprioceptive outputs are extracted. The second two panels show the output states for vision. The third and fourth panels respectively illustrate the prediction error for vision and the PB states.
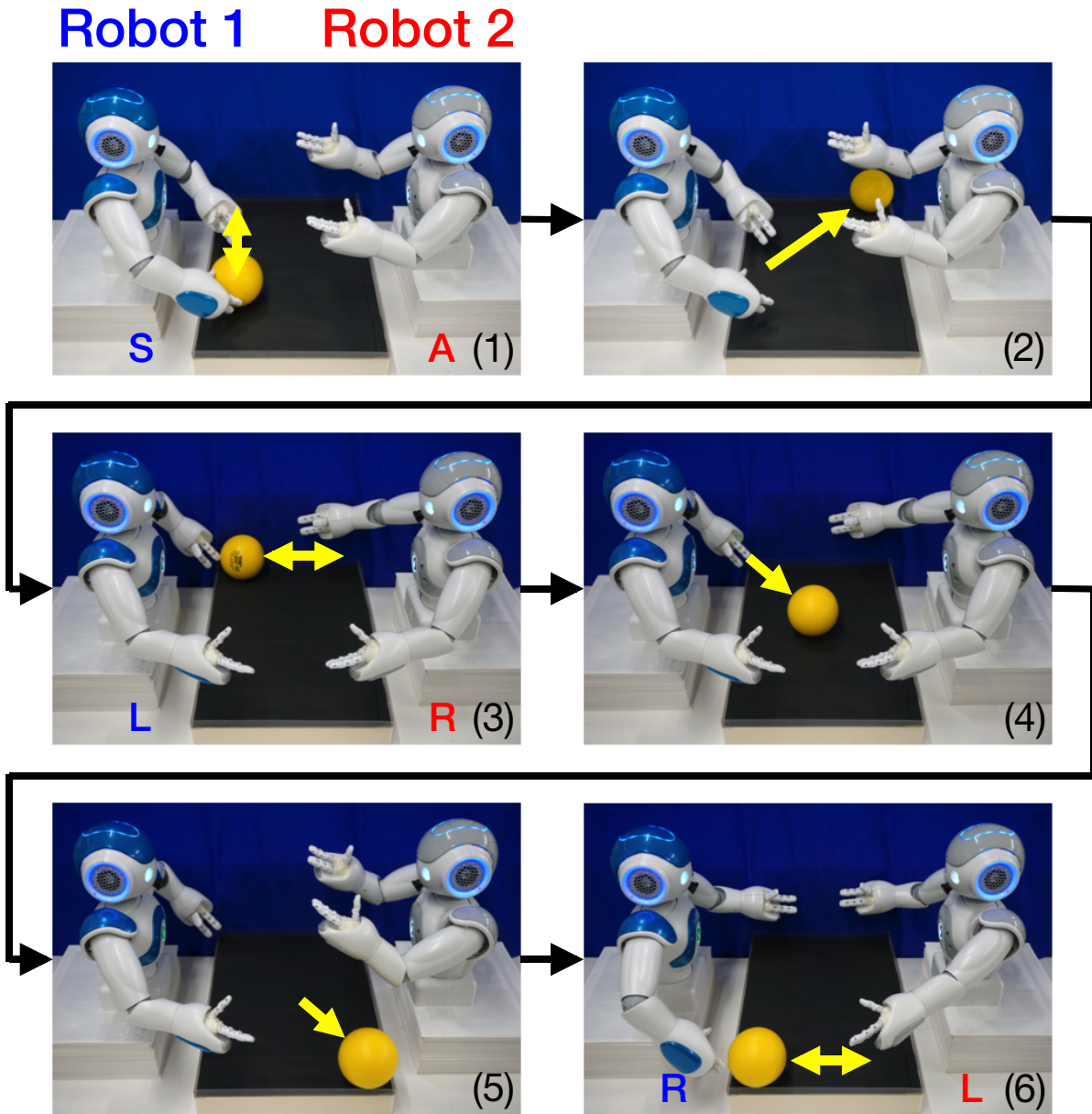
Figure 5. The actual process of interaction between Robot 1 (left) and Robot 2 (right). The phases (1), (3), and (6) show the period of performing interactive behaviors where the two robots completed the complementary tasks shown with the blue and red labels. The other phases (2), (4), and (5) show the transition period without performing specific behavioral patterns.

the workspace to another. When rolling to the other side, the ball must cross in front of the robots. Therefore, from 650 to around 800, Robot 2 attempted optimization to the correct PB, but the optimization was affected by previous input and thus modified to the wrong direction. Although Robot 2 failed to immediately optimize the PB states, it updated to the correct PB after a short time and adapted to the current situation. The deforming interaction was thus repaired and turned to the forming interaction. Although we show only a few possible behavior switches in Fig. 4, the robots successfully switched to all learning behavioral patterns in the experiment.

Figure 5 shows the actual process of interaction between the two robots mutually adapting to
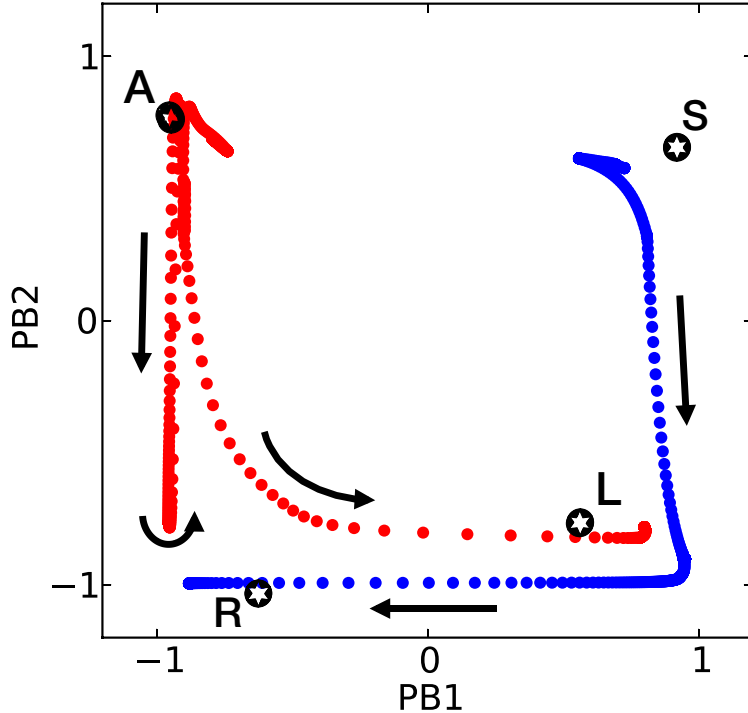
Figure 6. The PB states of two robots interacting during the generation phase. Black stars indicate learning results, and blue and red points respectively represent the generated PB states of Robot 1 and Robot 2. Black arrows show the direction of behavior transitions.

each other and switching their own behaviors to those relevant to the position of the manipulated object. Furthermore, when the two robots performed left (L) and right (R) behaviors, they attempted to synchronize their actions with the companion agent to complete complementary tasks. We thus observed correlation and coherent interaction between the robots. Figure 6 plots the PB states during the generation phase, and shows the transitions among the four learned behavioral patterns and that the modified PB dynamics were close to the learning results. Although forming and deforming of interactive behaviors spontaneously appeared, the PEM mechanism allowed the robots to react to changes with external causes. This phenomenon can be regarded as bidirectional interaction in which both agents influence each other.

5   Conclusion

We speculated that the PEM mechanism is essential for realizing the emergence of interactive behavior. We focused on external causes by using uncertain ball dynamics as a trigger for alternation of forming and deforming interactions in an unpredictable manner. We first tested the capability of the PEM mechanism via a robot controlled by a trained S-CTRNN model interacting with a human experimenter. Comparison of the experimental results between the case with and without the PEM mechanism indicated that the PEM mechanism is effective for the robot to adapt to the human movement in a unidirectional way. Then, the two robots experiments using identical dynamic neural network model was conducted to examine the effectiveness of the PEM mechanism in bidirectional adaptation which is required for achieving various types of social interaction between cognitive agents. The experimental results showed that complementary behaviors between the two robots can shift spontaneously among a set of trained ones as triggered by the potential instability in the physical ball interaction. It was

concluded that the top-down and the bottom-up interaction facilitated by the PEM mechanism can afford autonomous recovery from a particular interaction pattern once deformed by the external instability cause to newly formed interaction patterns for social cognitive agents.

This study used only ball position information for robot learning to address the problem of external causes generated by manipulated objects. Our future work will more comprehensively consider external causes to better deal with influences from the companion agent by including visual information about the partner's hands. We will also consider internal causes due to self-planning (e.g. ignoring others or preferring specific behaviors), as well as the influence of variance estimation contributing to the autonomous scaling of prediction error and the attention mechanism [24].

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