Comment on "Axial stretching and vortex definition" [Phys. Fluids 17, 038108 (2005)]

Pinaki Chakraborty and S. Balachandar
Department of Theoretical and Applied Mechanics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801

Ronald J. Adrian[a)]
Department of Mechanical and Aerospace Engineering, Arizona State University, Tempe, Arizona 85287

(Received 10 May 2005; accepted 2 August 2005; published online 21 February 2006)

[DOI: 10.1063/1.2169820]

In a recent paper, Wu, Xiong, and Yang[1] (hereafter referred to as WXY) use local flow kinematics to (i) explain the differences between the Q, Δ, and λ_2 vortex identification criteria and (ii) propose a general requirement mandatory for any definition of a vortex. Using Burgers and Sullivan vortices as analytical test cases, WXY compare the different vortex criteria based on their proposed vortex definition.

Our research group has been using local flow kinematics to understand the relationship between various local vortex identification schemes and our work is comprehensively reported in a recent paper[2] (hereafter referred to as CBA). Over the years this work at various stages of its development has been reported in a number of conferences and archival publications.[3–7] Both CBA and WXY use two local kinematic parameters,[8] λ_{ci} and $\lambda_{cr}/\lambda_{ci}$, to explore the relationships between the Q, Δ, and λ_2 criteria. These relationships are used to understand the differences between these criteria. In the interpretation of $\lambda_{cr}/\lambda_{ci}$, however, there is a fundamental difference between CBA and WXY: CBA use $\lambda_{cr}/\lambda_{ci}$ as a local measure of the orbital compactness of the instantaneous streamlines projected on the vortex plane, whereas WXY interpret $\lambda_{cr}/\lambda_{ci}$ to be a measure of the axial strain. WXY's interpretation is valid only in an incompressible flow, where the real eigenvalue satisfies the relation $\lambda_r = -2\lambda_{cr}$. This interpretation of $\lambda_{cr}/\lambda_{ci}$ is tied to their requirements for a vortex definition, to which we now focus attention.

WXY propose that any vortex definition should be independent of the axial strain. Using this definition, they analyze the Burgers and Sullivan vortices. They conclude that the Q and λ_2 criteria are deficient, since the vortex size extracted by them depends on the axial strain rate, γ. Based on the following scaling argument, we claim that this analysis and consequently the conclusion are flawed. For these test vortices, consider $\sqrt{\nu\gamma}$ and $\sqrt{\nu/\gamma}$ as velocity and length scales, respectively, to make the velocity field nondimensional. In a Burgers vortex, the velocity field becomes

$$v^*_{r^*} = -r^*/2, \quad v^*_\theta = \mathrm{Re}(1 - e^{-r^{*2}/4})/r^*, \quad v^*_{z^*} = -z^*, \qquad (1)$$

where $\mathrm{Re} = \Gamma/2\pi\nu$ is the vortex Reynolds number and the nondimensional quantities are denoted with an asterisk. Note that the γ dependence vanishes in the nondimensional velocity fields. Similarly, in the case of a Sullivan vortex, the above nondimensionalization results in the corresponding velocity fields becoming independent of γ. Hence, the vortex sizes (scaled by $\sqrt{\nu/\gamma}$)[9] educed by the different criteria cannot depend on γ. Additionally, $\lambda_{cr}/\lambda_{ci}$ also does not depend on γ. Therefore, the analysis and all the figures in WXY are misleading.[10]

Now consider WXY's requirement that any vortex definition should be independent of axial strain in the context of an incompressible flow, where the axial strain is related to the rate of spiraling in or out on the plane of the vortex ($\lambda_{cr}/\lambda_{ci}$). Their proposal translates to the following criterion for vortex identification (which they call the $Q_{2\mathrm{D}}$ criterion): $\lambda_{ci}^2 = Q_{2\mathrm{D}} > 0$ with no restriction on the ratio $\lambda_{cr}/\lambda_{ci}$. In contrast, the Q and λ_2 criteria limit the admissible range of $\lambda_{cr}/\lambda_{ci}$. For example, $Q > 0$ is equivalent to $\lambda_{ci} > 0$ and $|\lambda_{cr}/\lambda_{ci}| < 1/\sqrt{3}$. It may be noted that the $Q_{2\mathrm{D}} > 0$ criterion is identical to the $\Delta > 0$ criterion or the "swirling strength" $\lambda_{ci} > 0$ criterion.[3] In fact, since $Q_{2\mathrm{D}} = \lambda_{ci}^2$, the $Q_{2\mathrm{D}}$ criterion is identical to the λ_{ci} criterion even at nonzero thresholds. The λ_{ci} criterion and the Δ criterion, however, are equivalent only at zero threshold and they differ at nonzero thresholds.[2]

The requirement of vortex definition being independent of $\lambda_{cr}/\lambda_{ci}$ is not appropriate. The ratio $\lambda_{cr}/\lambda_{ci}$ measures the spatial compactness of material points as they swirl around. In order for the notion of a vortex to be useful from dynamical and statistical perspectives, the material points within the vortex should remain close as the vortex continues to evolve in time.[2,11] For example, Fig. 1 of CBA clearly illustrates how, irrespective of local swirling strength, when $\lambda_{cr}/\lambda_{ci}$ takes a large positive value, the instantaneous streamlines spiral radially out so rapidly that they cannot be classified as part of a vortex core. In order to qualify as vortex core, it is appropriate to require a threshold for the ratio $\lambda_{cr}/\lambda_{ci}$. Thus WXY's assertion that the $Q_{2\mathrm{D}}$ criterion is better than Q and λ_2 criteria is inaccurate. In fact, as suggested by CBA, it is best to consider both the parameters λ_{ci} and the ratio $\lambda_{cr}/\lambda_{ci}$, and choose their thresholds appropriately as dictated by the length and time scales of the problem.

In addition, WXY have other shortcomings: (i) Their observation that $\lambda_2 < 0$ imposes stricter restriction for shrinking ($\lambda_{cr}/\lambda_{ci} < 0$) than stretching ($\lambda_{cr}/\lambda_{ci} > 0$) is incorrect, be-

[a)]Also at Department of Theoretical and Applied Mechanics, University of Illinois at Urbana-Champaign, Urbana, IL 61801.

cause owing to its formulation, λ_2 is invariant to the sign of $\boldsymbol{\nabla v}$. That is, positive or negative $\boldsymbol{\nabla v}$ imply the same λ_2, hence the region of $\lambda_2 < 0$ is invariant to the sign of $\lambda_{cr}/\lambda_{ci}$. (ii) WXY mention the sign of Q_{2D}, while by definition Q_{2D} is non-negative. (iii) Finally, the affine transformation matrix $\boldsymbol{P}$ defined in WXY should contain the real and imaginary parts of the complex conjugate eigenvector, as well as the real eigenvector.

In conclusion, we have shown that WXY's conclusion from analysis of the two vortex examples in WXY is flawed, that in CBA and WXY there exist similarities in the approach but serious differences in the interpretation, and that WXY's general requirements for any vortex definition need further justification.

[1]J.-Z. Wu, A.-K. Xiong, and Y.-T. Yang, "Axial stretching and vortex definition," Phys. Fluids **17**, 038108 (2005).

[2]P. Chakraborty, S. Balachandar, and R. J. Adrian, "On the relationships between local vortex identification schemes," J. Fluid Mech. **535**, 189 (2005).

[3]J. Zhou, R. J. Adrian, S. Balachandar, and T. M. Kendall, "Mechanisms for generating coherent packets of hairpin vortices," J. Fluid Mech. **387**, 353 (1999).

[4]P. Chakraborty, S. Balachandar, and R. J. Adrian, "Vortex identification in turbulent flows: isotropic, sphere wake," in Proceedings of 4th ASE/JSME Joint Fluids Engineering Conference, Hawaii, July 2003, FEDSM2003–45082, CDROM.

[5]P. Chakraborty, S. Balachandar, and R. J. Adrian, "Toward a unified local vortex identification criterion," Bull. Am. Phys. Soc. **48** (10), 163 (2003).

[6]P. Chakraborty, S. Balachandar, and R. J. Adrian, "On local vortex identification," extended summary in Proceedings of the 21st ICTAM, Warsaw, Poland, August 2004, FM25_12161, CDROM.

[7]P. Chakraborty, S. Balachandar, and R. J. Adrian, "Local vortex identification criteria: inter-relationships and a unified outlook," in *Fluid Mechanics and Its Applications, IUTAM Symposium on Elementary Vortices and Coherent Structures: Significance in Turbulence Dynamics*, Proceedings of IUTAM Conference in Kyoto, Japan, October 2004 (Springer, New York, 2005), Vol. 79, p. 111.

[8]We denote the real and imaginary components of the complex conjugate eigenvalues of the velocity gradient tensor $\mathbf{\nabla v}$ by λ_{ci} and λ_{cr}, respectively. WXY use the notation σ_r and σ_i for λ_{ci} and λ_{cr}, respectively, and k (which they refer to as the axial strain ratio) for $\lambda_{cr}/\lambda_{ci}$. These local kinematic parameters λ_{ci} and λ_{cr} have an unambiguous physical interpretation: in the vortex plane, λ_{ci} is a local measure of the strength of the swirling and $\lambda_{cr}/\lambda_{ci}$ is a local measure of the compactness of the spiraling orbits (refer to CBA for details). The vortex plane is identified to be the plane spanned by the real and imaginary components of the complex conjugate eigenvector.

[9]For both the vortices, the vortex size scales with $\sqrt{\nu/\gamma}$. For example, in a Burgers vortex, the vortex size is $2.24\sqrt{\nu/\gamma}$ and it corresponds to the location of the maximum of tangential velocity. Hence the question of interest is the dependence of the nondimensional vortex size on γ.

[10]For example, Fig. 1 of WXY ignores the fact that $Q_{2D} \sim \gamma^2$. With this dependence, γ^2 factors out in Eqs. (17a)–(17c) of WXY for the $\Delta > 0$, $Q > 0$, and $\lambda_2 < 0$ criteria. Hence the vortex size extracted by all these criteria are invariant to γ, contrary to what is depicted in Fig. 1 of WXY. It may be noted that in contrast with the $\Delta > 0$ criterion, the vortices educed by $Q > 0$ and $\lambda_2 < 0$ criteria depend on the vortex Reynolds number $\mathrm{Re} = \Gamma/2\pi\nu$ (see CBA for details).

[11]R. Cucitore, M. Quadrio, and A. Baron, "On the effectiveness and limitations of local criteria for the identification of a vortex," Eur. J. Mech. B/Fluids **18**, 261 (1999).